Chapter 9 – Slice Selection and Multidimensional Imaging

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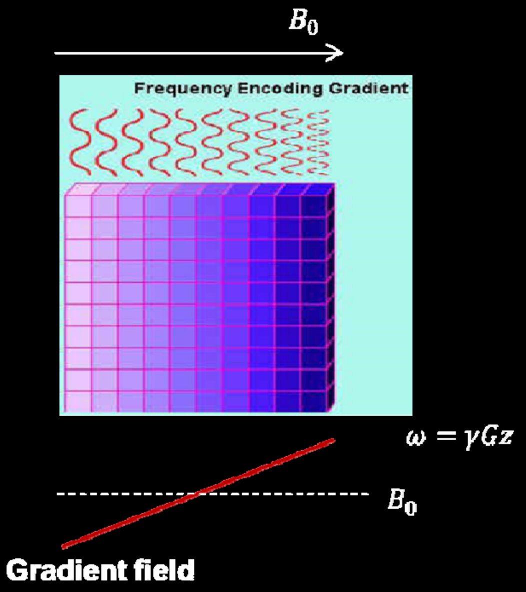
Quick Recap:

$$S(t) = \Lambda \cdot \omega_0 \cdot \beta_\perp \int d^3r \cdot M_\perp(\vec{r},0) \cdot e^{i(\Omega-\omega(\vec{r},t))t}$$

$$S(t) = \int d^3r \cdot \rho(\vec{r}) \cdot e^{i(\Omega-\omega(\vec{r},t))t}$$

$$\rho(\vec{r}) = \Lambda \cdot \omega_0 \cdot \beta_\perp \cdot M_0(\vec{r}) = \Lambda \cdot \omega_0 \cdot \beta_\perp \cdot \rho_0(\vec{r}) \frac{\gamma^2 \cdot \hbar^2 \cdot B_0}{4 \cdot k \cdot T}$$

… the effective spin density



$$B(z) = B_0 + G \cdot z$$

$$\boxed{\omega(z) = \omega_0 + \gamma \cdot G \cdot z}$$

$$\phi(\vec{r},t) = \int_0^t (\Omega - \omega(\vec{r},t')) \cdot dt'$$

$$\Omega = \omega_0 \qquad \omega(\vec{r},t) = \omega_0 + \Delta\omega(\vec{r},t) \qquad \omega(z) = \omega_0 + \gamma \cdot G \cdot z$$

$$\varphi(\vec{r},t) = \int_0^t -\Delta\omega(\vec{r},t') \cdot dt' \qquad \phi(z,t) = \int_0^t -\gamma \cdot G \cdot z \cdot dt'$$

$$\Rightarrow \phi(z,t) = -\gamma \cdot G \cdot z \cdot t$$

$$\boxed{S(t) = \int dz \cdot \rho(z) \cdot e^{-i \cdot \gamma \cdot G \cdot z \cdot t}}$$

Quick Recap:

$$S(t) = \int dz \cdot \rho(z) \cdot e^{-i \cdot \gamma \cdot G \cdot z \cdot t}$$

$$\phi(z,t) = 2\pi \cdot k \cdot t \qquad k = \frac{\gamma}{2\pi} \cdot G \cdot t \qquad k = \bar{\gamma} \cdot G \cdot t \qquad \bar{\gamma} \equiv \cancel{\gamma}$$

$$\boxed{S(t) = S(k) = \int dz \cdot \rho(z) \cdot e^{-i \cdot 2\pi \cdot k \cdot z}}$$

… the 1D imaging equation

$$k(t) = \int_0^t \bar{\gamma} \cdot G(t) \cdot dt' \qquad \phi(z,t) = -2\pi \cdot k(t) \cdot z$$

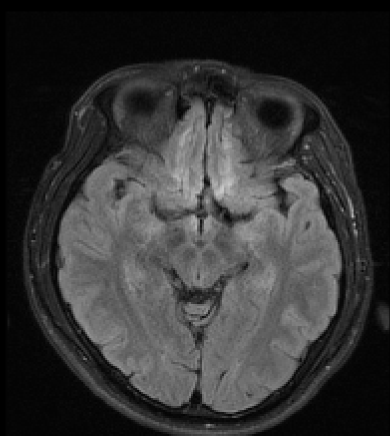
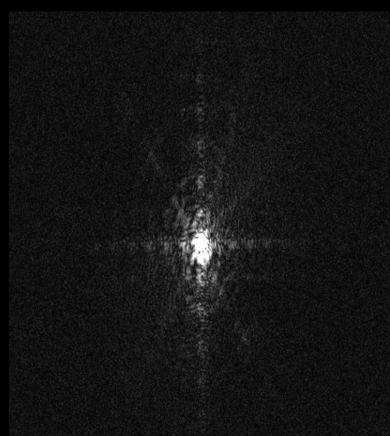


Image space



k-space

Units of k variable…

$$\frac{rad}{sec \cdot Tesla} \cdot \frac{1}{rad} \cdot \frac{Tesla}{meter} \cdot sec \equiv \frac{1}{meter}$$

Quick Recap:

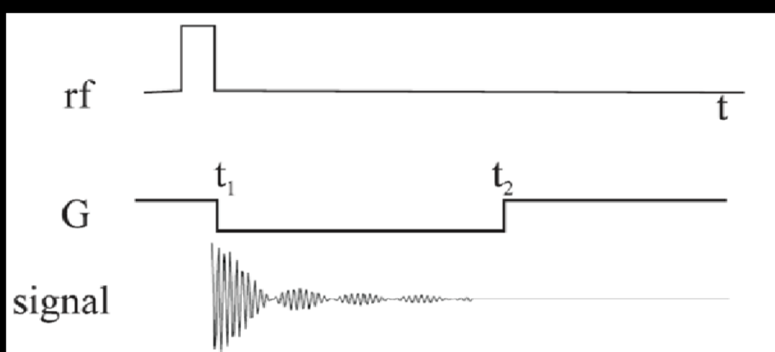
Fourier Transform Pair

$$S(t) = S(k) = \int_{-\infty}^{+\infty} dz \cdot \rho(z) \cdot e^{-i \cdot 2\pi \cdot k \cdot z}$$

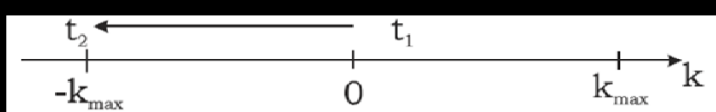
Inverse Fourier Transform…

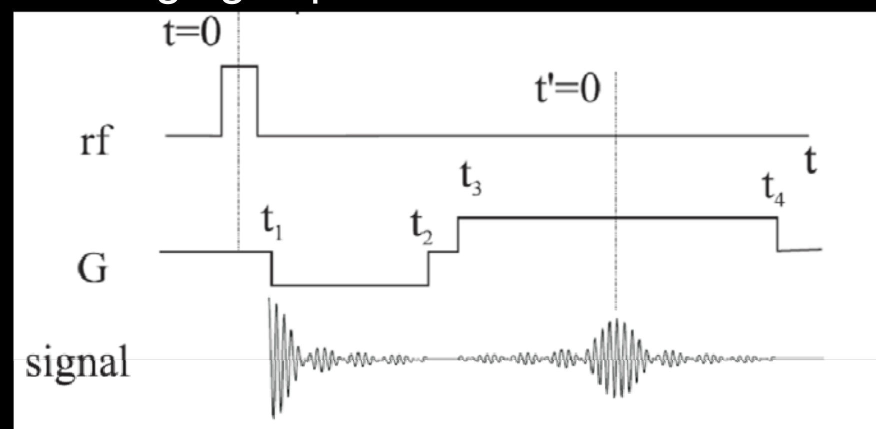
$$\rho(z) = \int_{-\infty}^{+\infty} dk \cdot S(k) \cdot e^{+i \cdot 2\pi \cdot k \cdot z}$$

1D imaging experiment with Gradient Echo

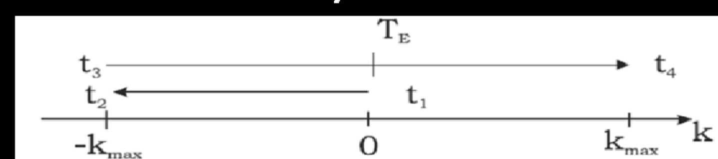


$k = \bar{\gamma} \cdot -G \cdot t$ $t_1 < t < t_2$



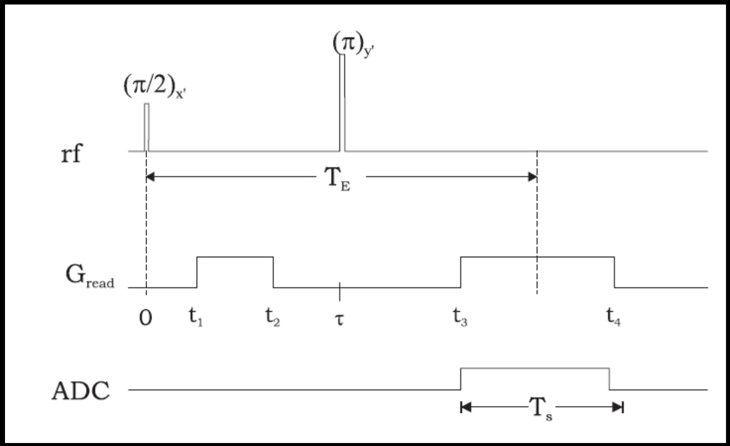


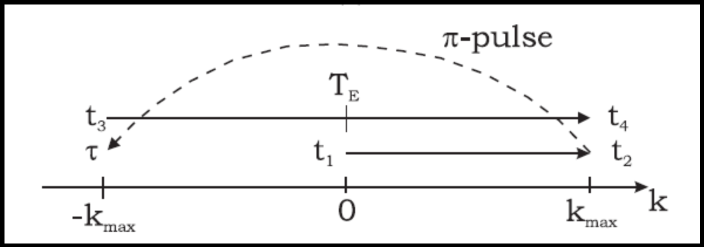
$k = \bar{\gamma} \cdot G \cdot t$ $t_3 < t < t_4$

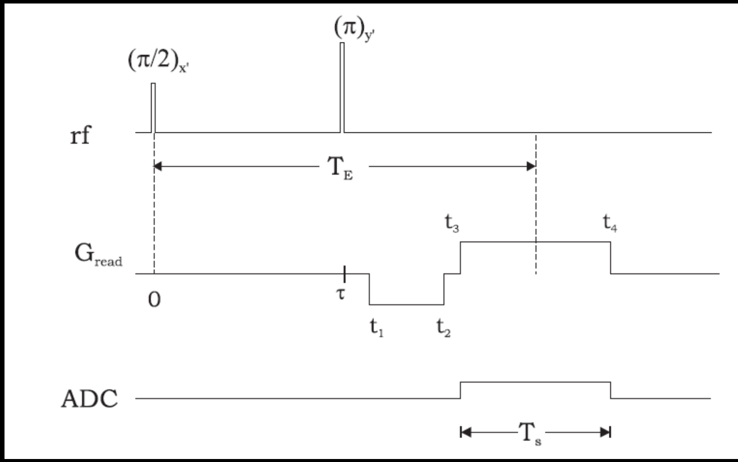


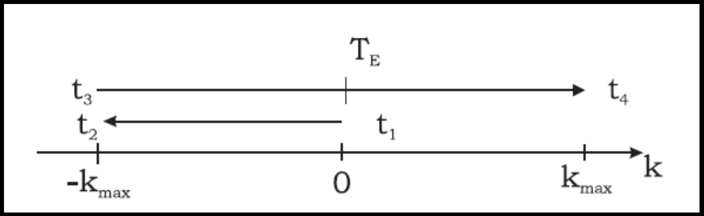
Quick Recap:

1D imaging experiment using a Gradient Echo coinciding with the Spin Echo

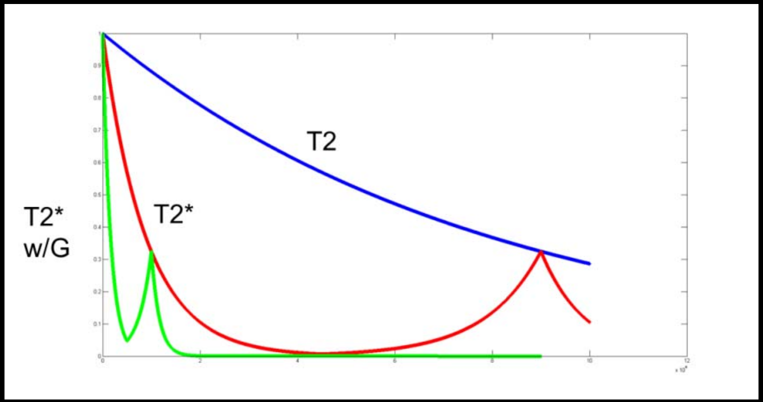








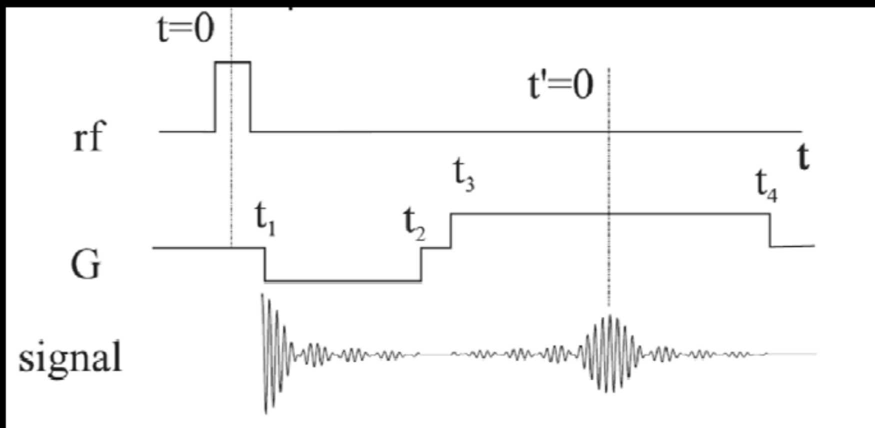
Sampling time was denoted as T_s



Slice Selection and Multidimensional Imaging

- Slice excitation and Bandwidth
- Phase encoding (2D imaging)
- 3D imaging
- k-space trajectories

Imaging Equation



$$S(t) = \int dz \cdot \rho(z) \cdot e^{-i \cdot \gamma \cdot G_z \cdot z \cdot t}$$

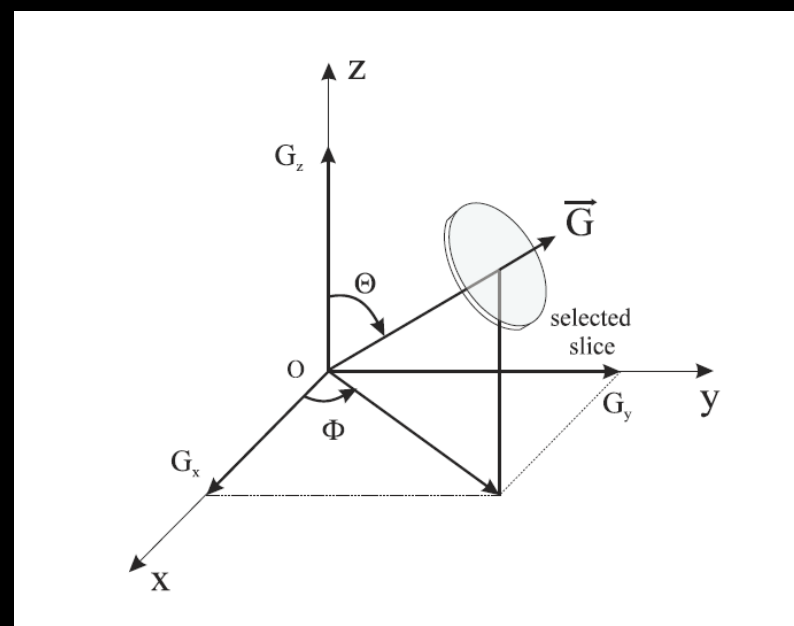
$$\boldsymbol{k_z}(t) = \int_0^t \bar{\gamma} \cdot G_z(t) \cdot dt' \qquad \phi(z, t) = -2\pi \cdot k_z(t) \cdot z$$

$$S(t) = S(k_z) = \int dz \cdot \rho(z) \cdot e^{-i \cdot 2\pi \cdot \boldsymbol{k_z} \cdot z}$$

$$\begin{aligned} \vec{G}(t) &\equiv \vec{\nabla} B_z^g(\vec{r}) \\ &= \hat{x}\frac{\partial}{\partial x}B_z^g + \hat{y}\frac{\partial}{\partial y}B_z^g + \hat{z}\frac{\partial}{\partial z}B_z^g \\ &\equiv G_x(t)\hat{x} + G_y(t)\hat{y} + G_z(t)\hat{z} \end{aligned}$$

$$\phi(\vec{r}, t) = -\gamma \vec{r} \cdot \int_0^t dt' \vec{G}(t') \qquad \vec{k}(t) = -\bar{\gamma} \int_0^t dt' \vec{G}(t')$$

$$s(\vec{k}) = \int d^3 r \rho(\vec{r}) e^{-i 2\pi \vec{k} \cdot \vec{r}}$$



Imaging Equation

1D: $s(k_x) = \int dx \cdot \rho(x) \cdot e^{-i \cdot 2 \cdot \pi \cdot k \cdot x}$

$x \to \vec{r} = (x, y, z)$

3D: $s(k) = \int d^3 r \cdot \rho(r) \cdot e^{-i2\pi(\vec{r}\cdot\vec{k})}$

1) All dimensions are equivalent
2) k along each direction is determined by the gradient in that direction

$$s(k_x, k_y, k_z) = \iiint dx \cdot dy \cdot dz \cdot \rho(x, y, z) \cdot e^{-i2\pi(k_x x + k_y y + k_z z)}$$

$$k_x(t) = \gamma \int^t G_x(t')dt', \quad k_y(t) = \gamma \int^t G_y(t')dt', \quad k_z(t) = \gamma \int^t G_z(t')dt'$$

Imaging Equation

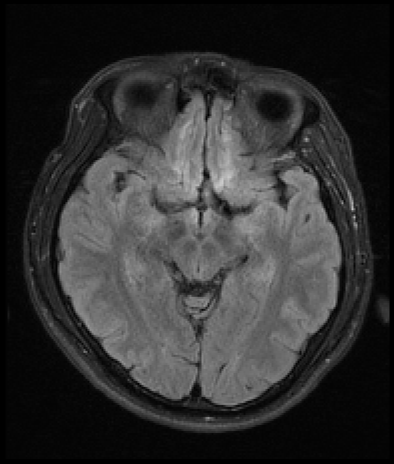
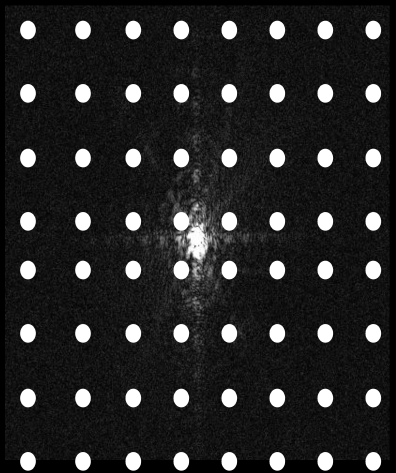


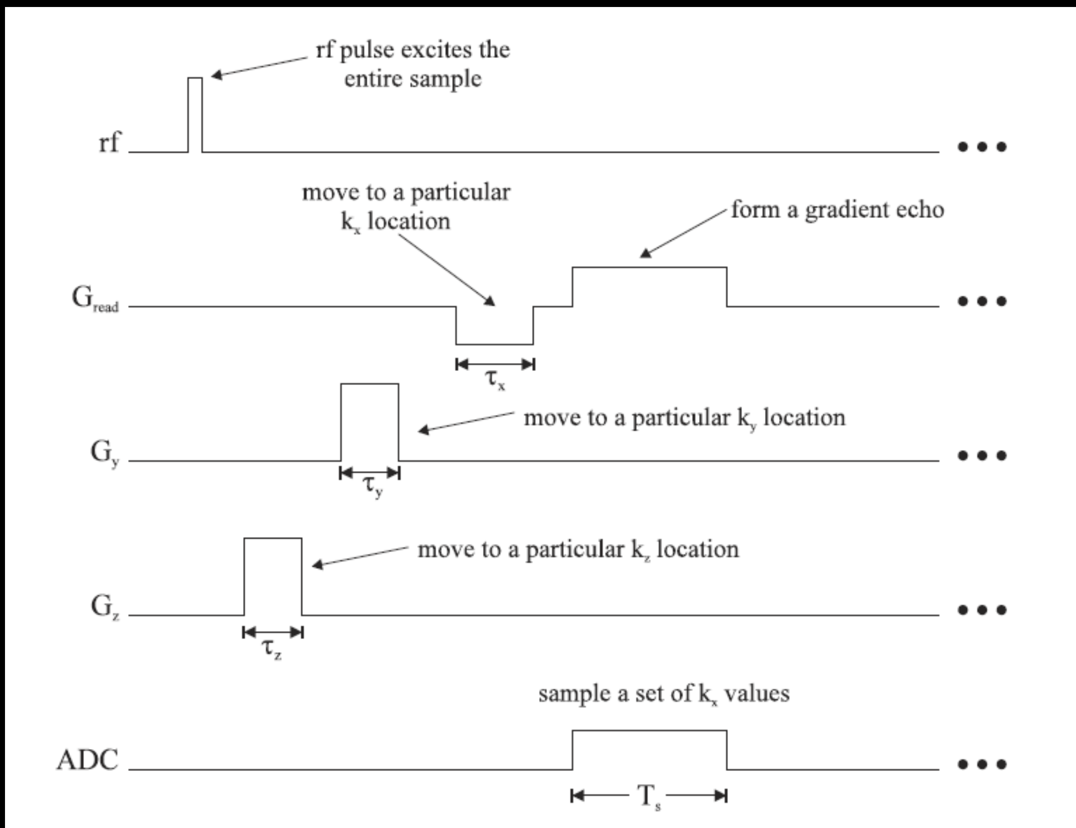
Image space

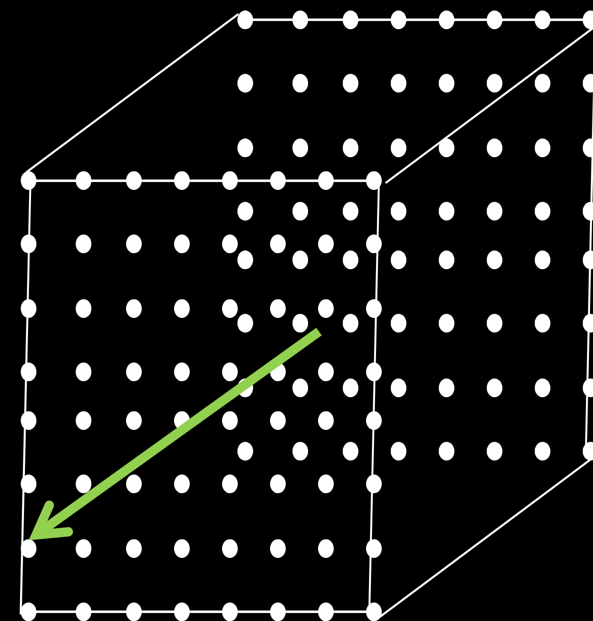


k-space

Filling up the corresponding 2D/3D k-space to obtain the image

Multidimensional Spatial Encoding (2D/3D imaging)

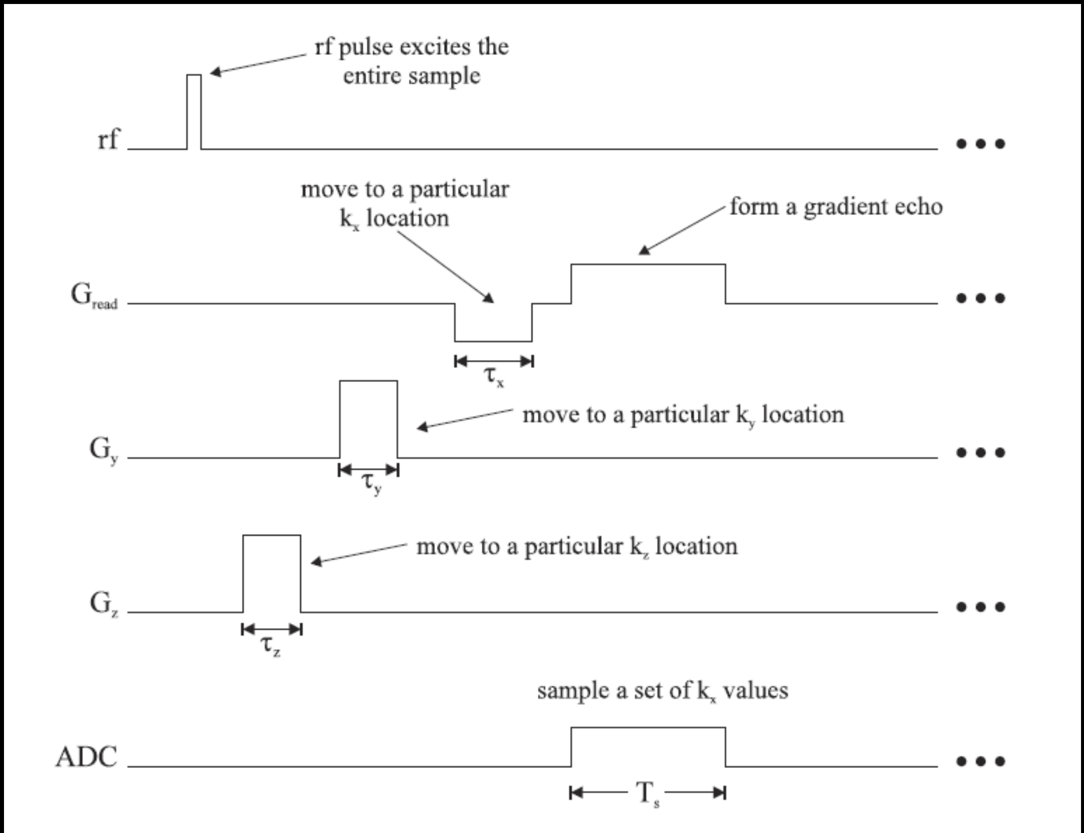


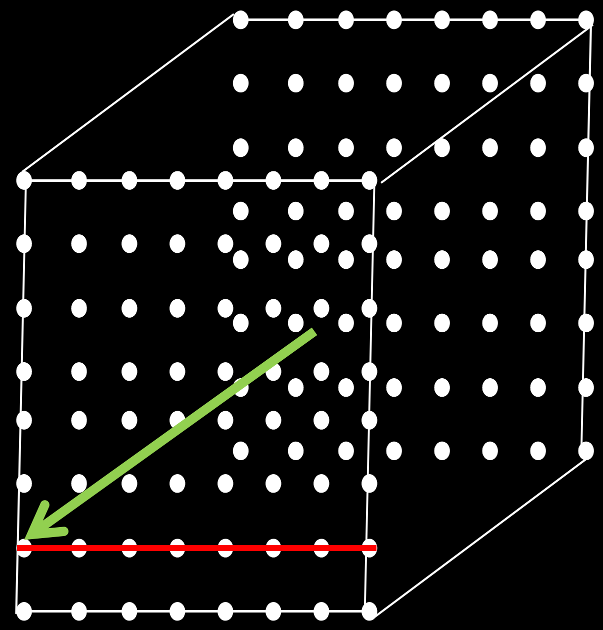


$$k_y = \bar{\gamma} \cdot G_y \cdot \tau_y$$

$$k_x(t) = \gamma \int^t G_x(t')dt', \quad k_y(t) = \gamma \int^t G_y(t')dt', \quad k_z(t) = \gamma \int^t G_z(t')dt'$$

Multidimensional Spatial Encoding (2D/3D imaging)

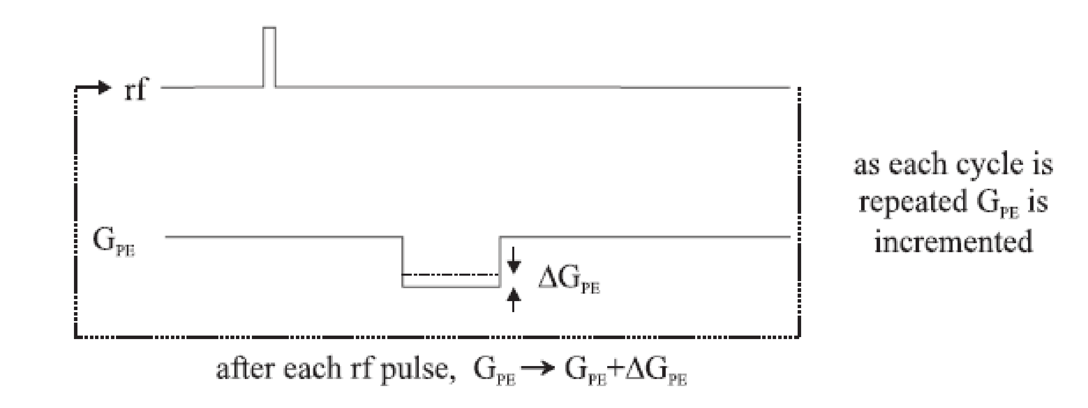




$$k_y = \bar{\gamma} \cdot G_y \cdot \tau_y$$

$$k_x(t) = \gamma \int^t G_x(t')dt', \quad k_y(t) = \gamma \int^t G_y(t')dt', \quad k_z(t) = \gamma \int^t G_z(t')dt'$$

Phase encoding

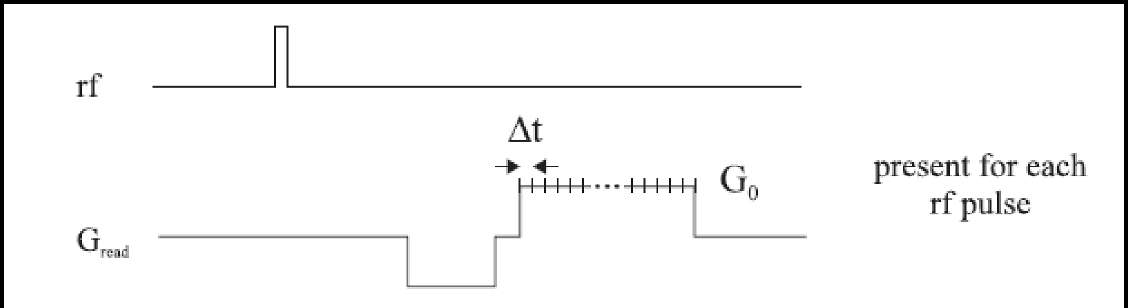


Phase encoding

$$\Delta k_y = -\bar{\gamma} \cdot \Delta G_y \cdot \tau$$

Encoding phase is accumulated in steps of the phase encoding gradient

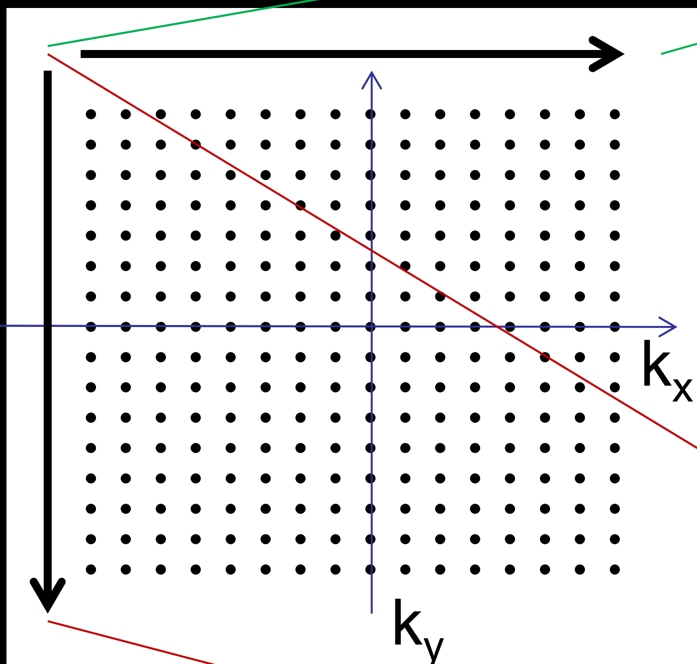
Frequency encoding



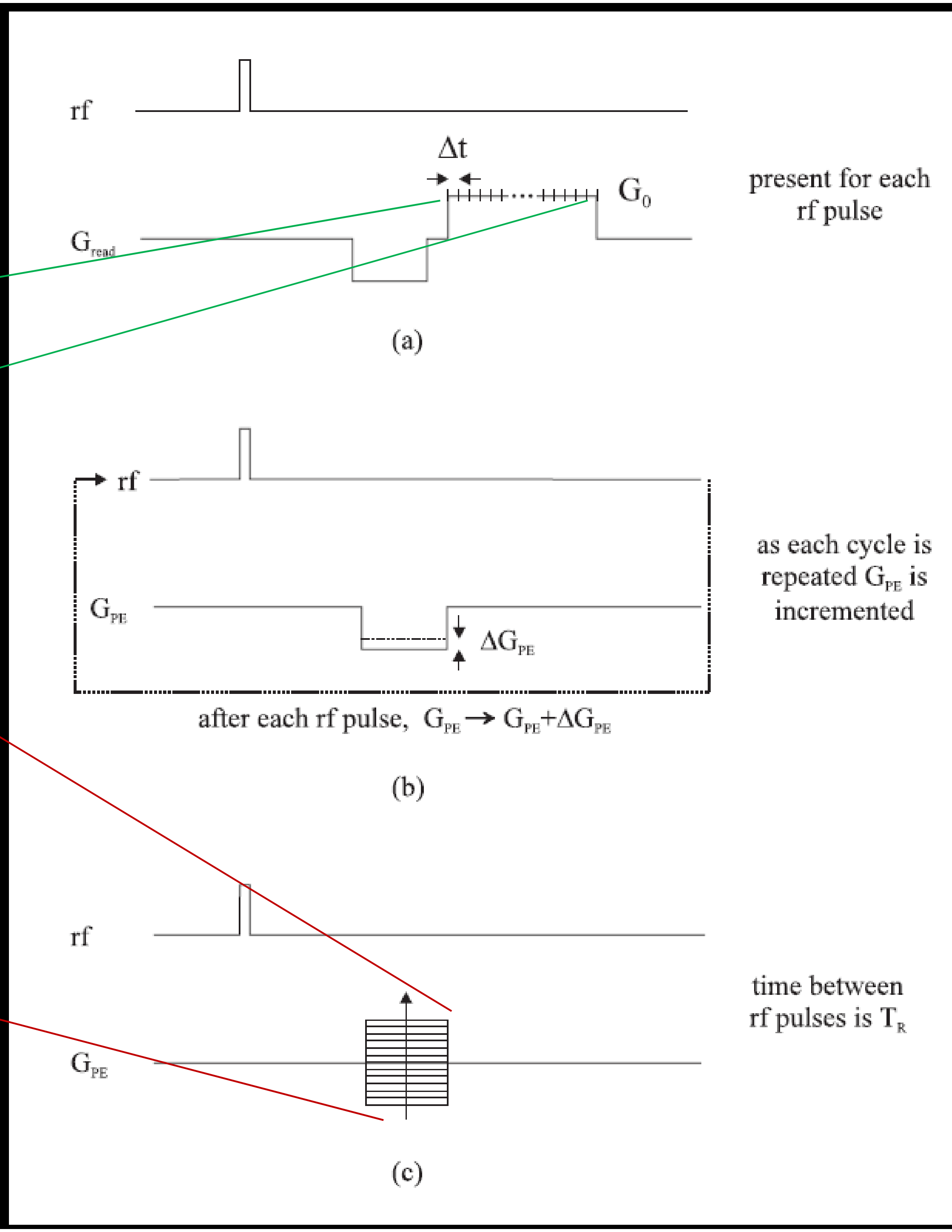
$$k_x = \gamma \cdot G_x \cdot t = \gamma \cdot G_x \cdot \Delta t$$

Encoding phase is accumulated with the progression of time

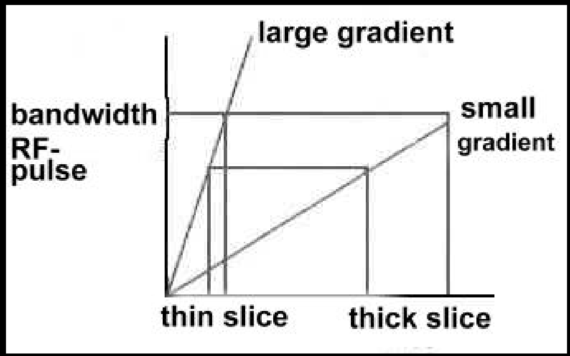
Phase encoding

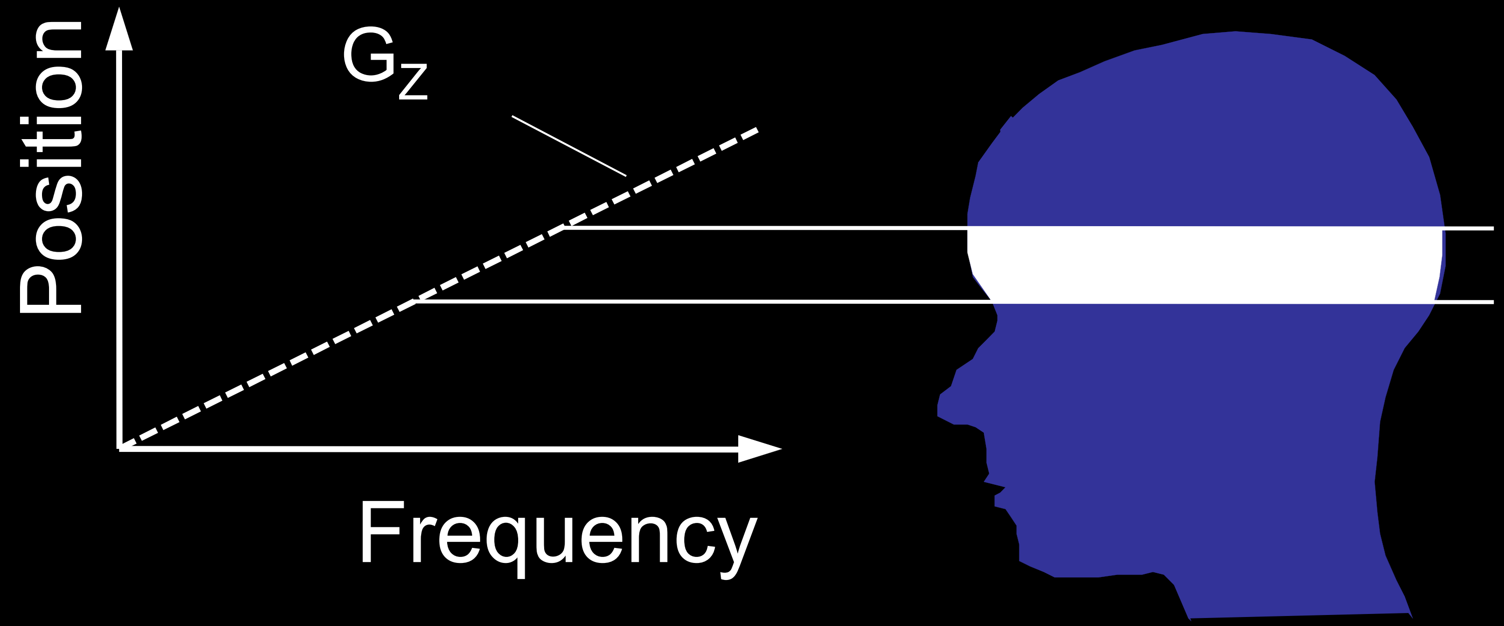


$$k_y = \gamma \cdot G_y \cdot t$$
$$= \gamma \cdot \Delta G_y \cdot \tau$$

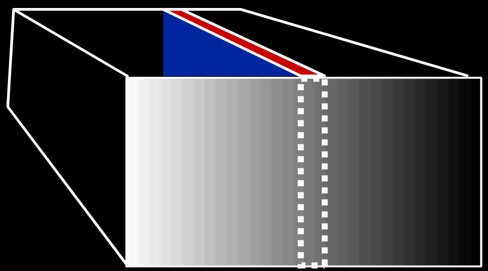


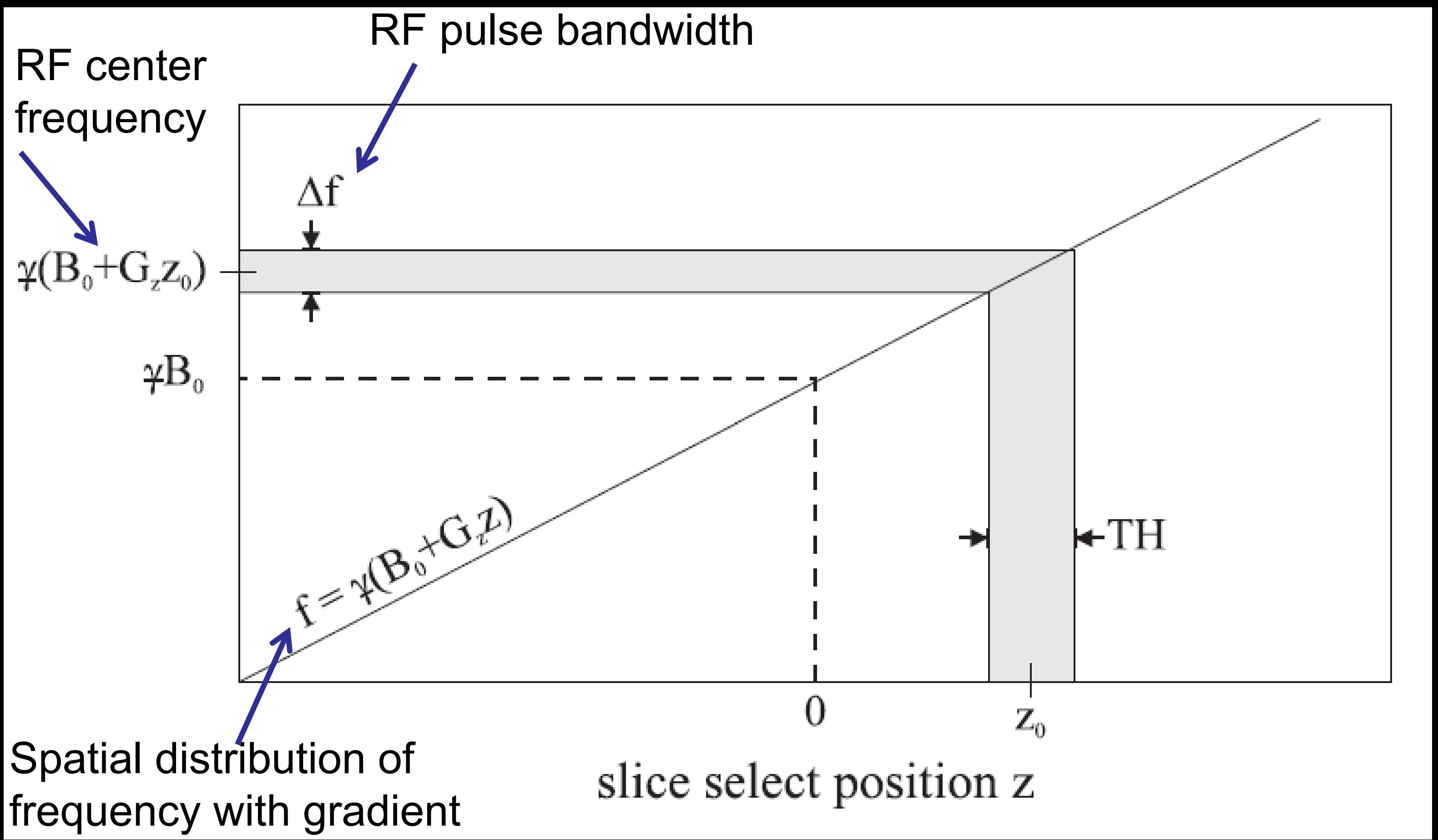
Slice Selection





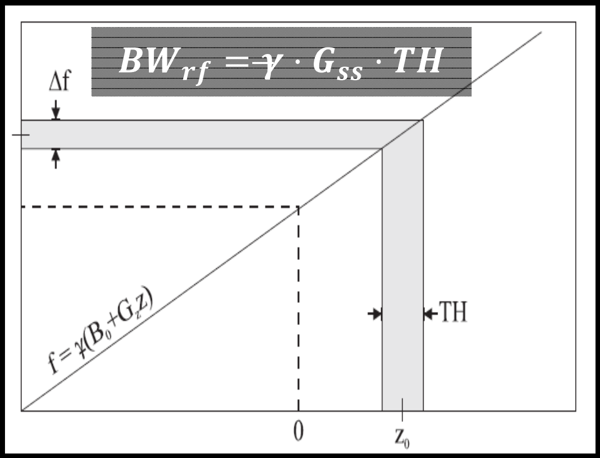
Slice selection

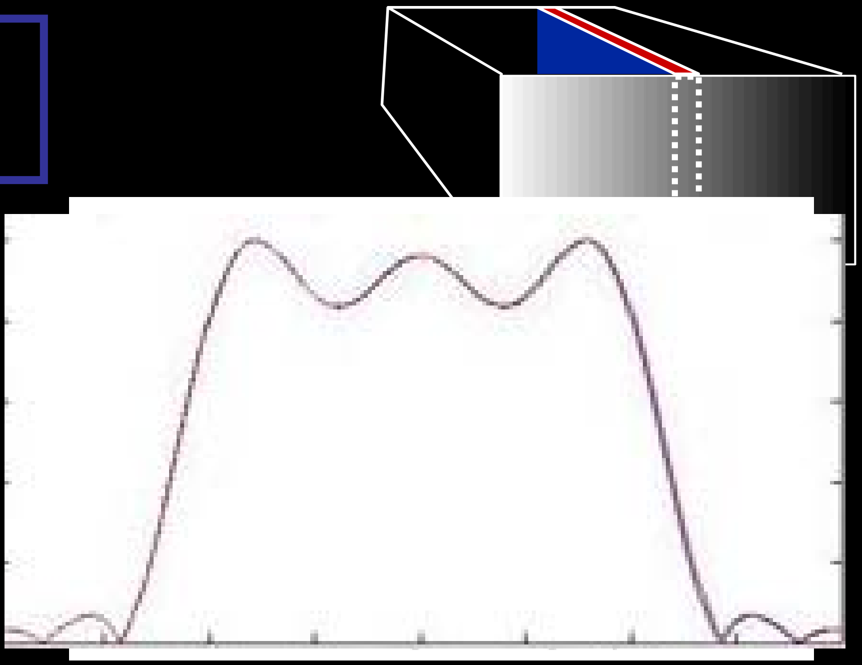




Slice selection

Slice selection

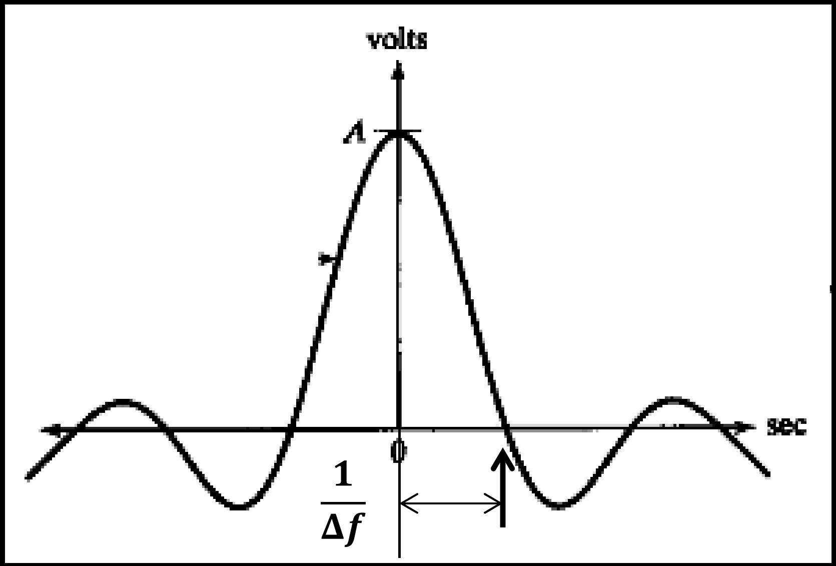




$$\Delta f = BW_{rf}$$

$$\tau_{rf} = n_{zc} \cdot \left(\frac{1}{BW_{rf}}\right)$$

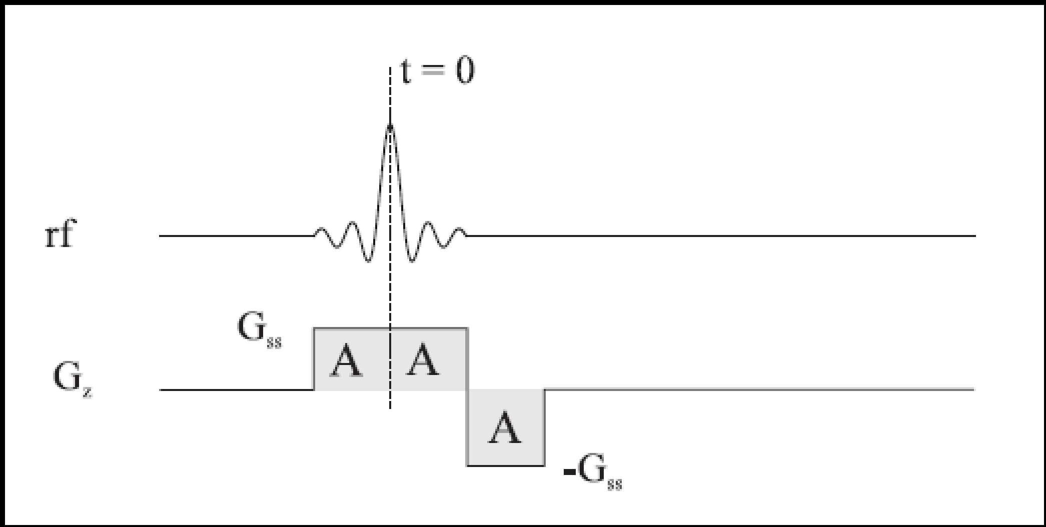
$$BW_{rf} = \Delta f = \gamma \cdot G_{ss} \cdot TH$$



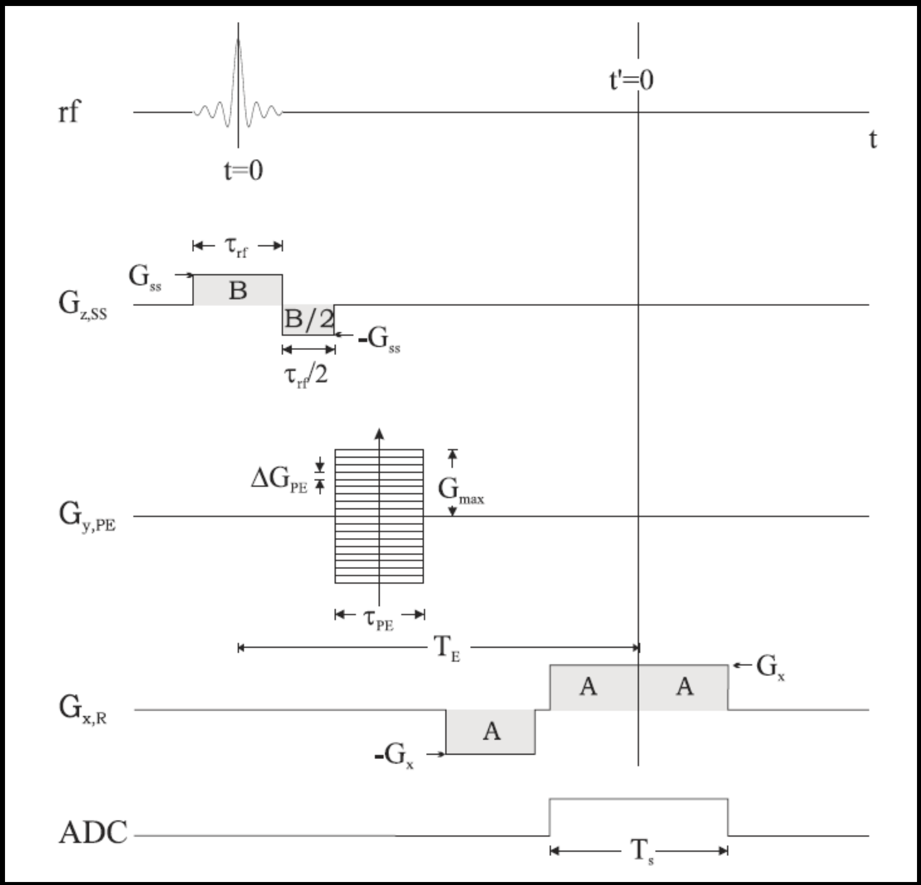
Slice selection gradient refocusing

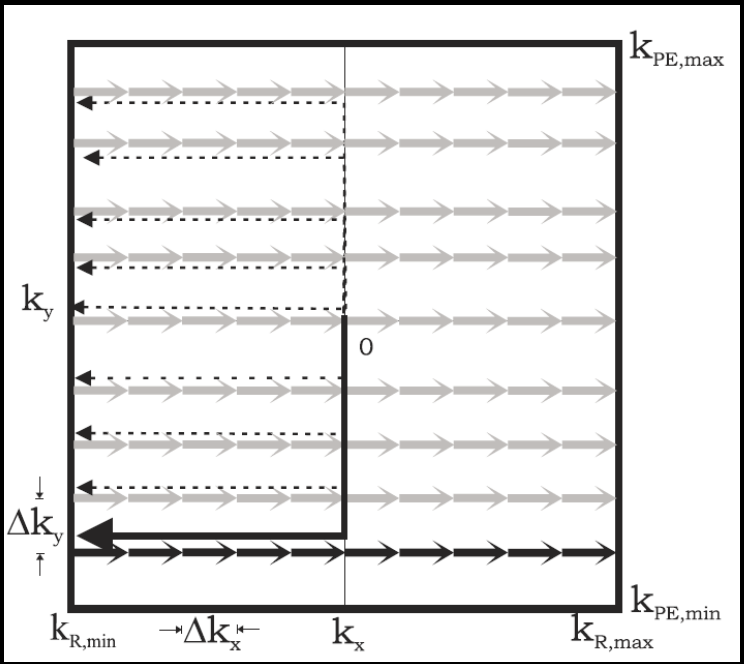
Spin dephasing during excitation

Spins are considered instantaneously tipped at the center of RF pulse. So, they start getting dephased under the influence of G_{ss} (1/2)



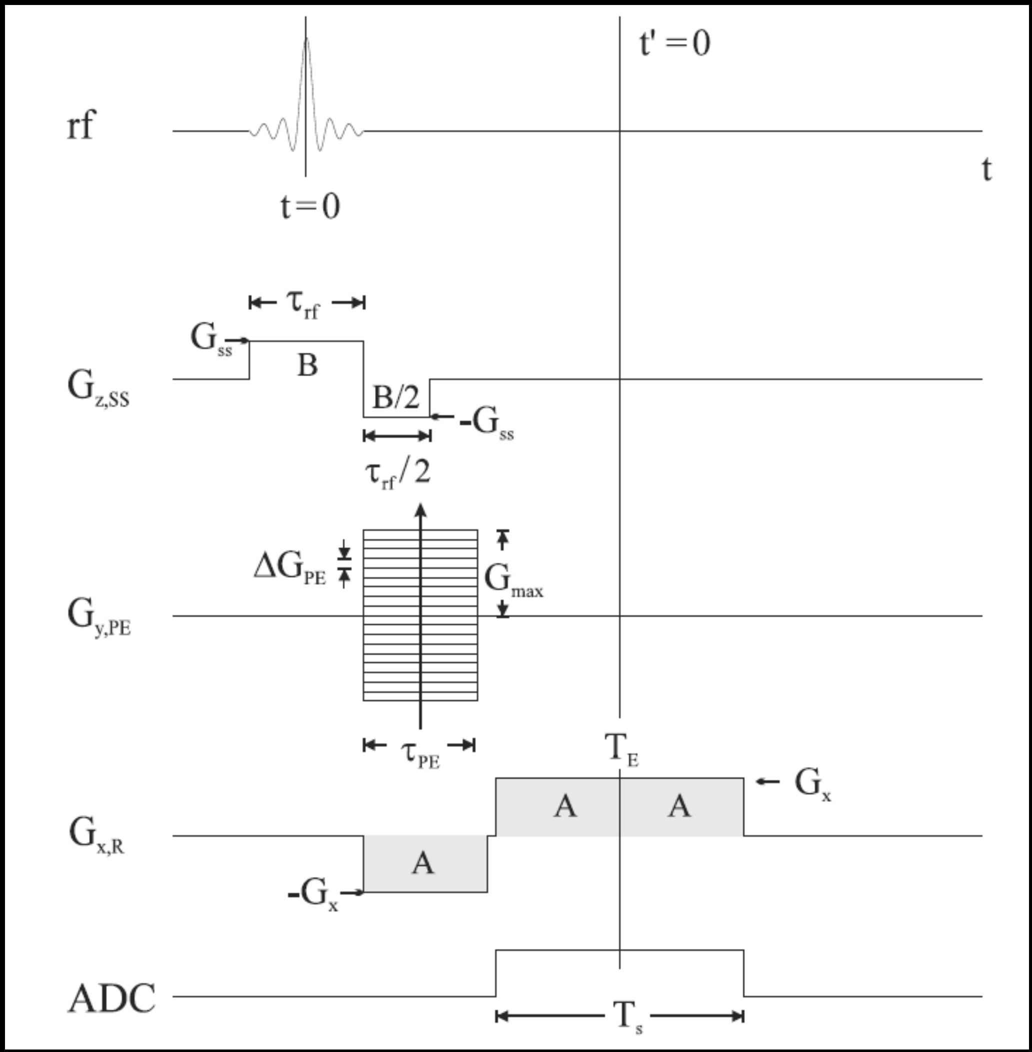
2D Imaging sequence

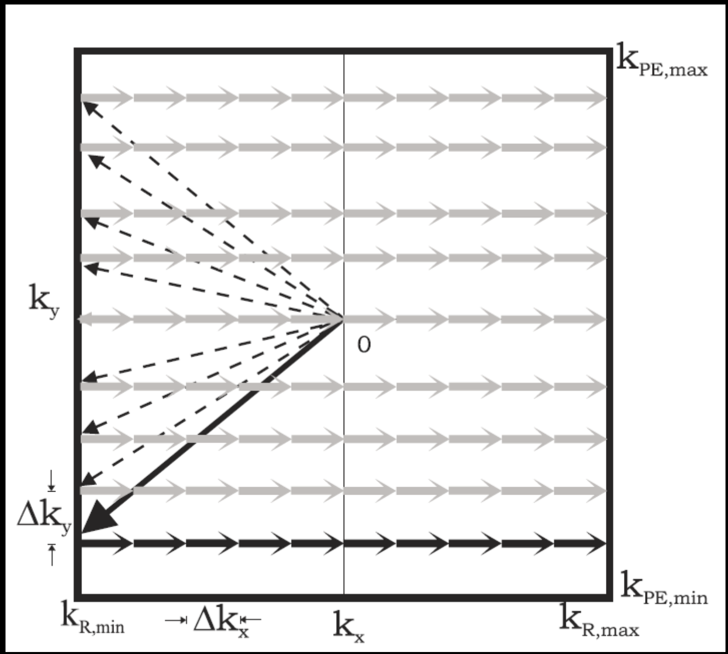




$$k_{read\_min} = \bar{\gamma} \cdot -G_{read} \cdot \frac{T_s}{2}$$

2D Imaging sequence

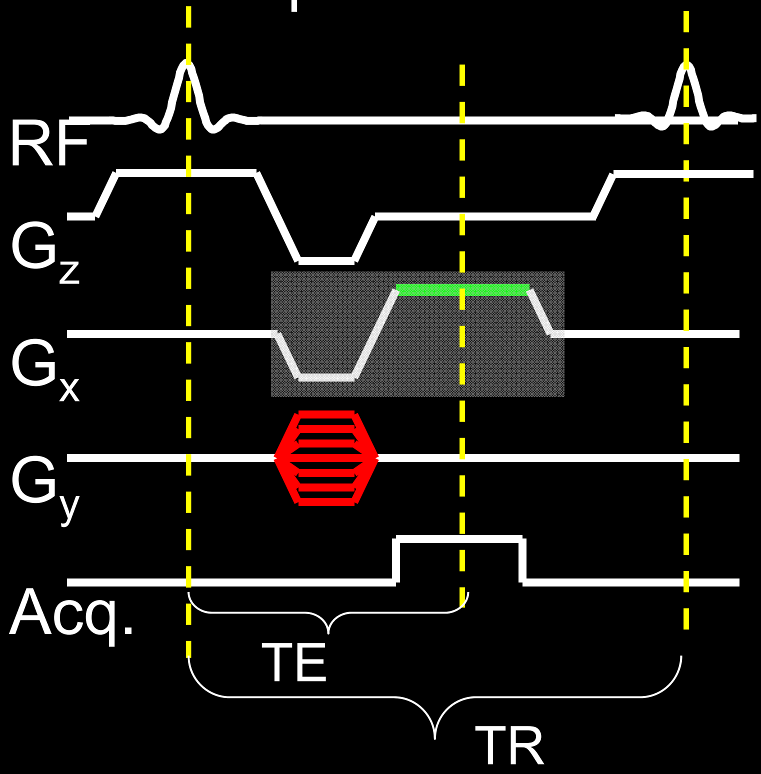


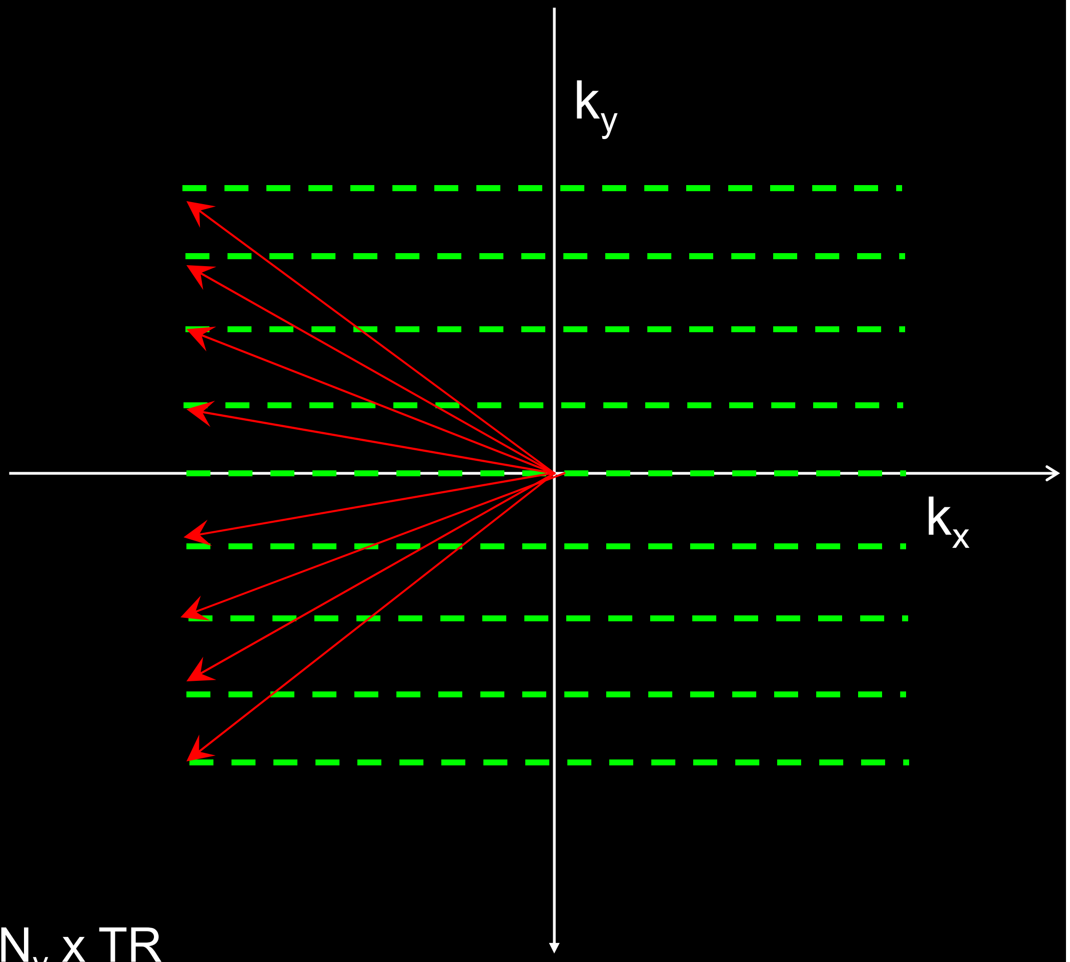


$$k_{read\_min} = \bar{\gamma} \cdot -G_{read} \cdot \frac{T_s}{2}$$

2D imaging: K-space trajectory

Typical Gradient
Echo sequence



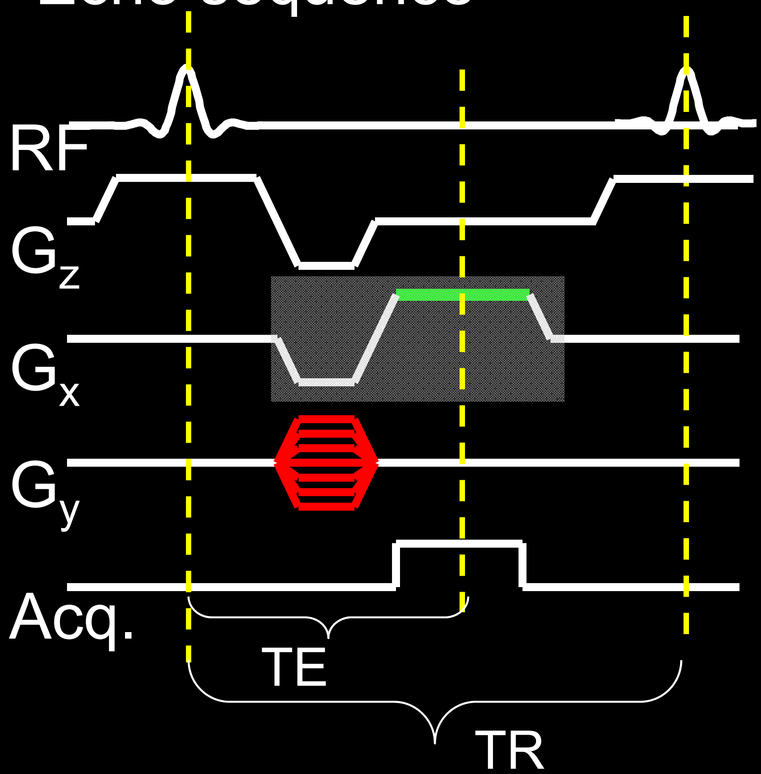


Total imaging time for a single slice –N_y x TR

Sampling of the signal line by line forming the image in
Spatial frequency domain

2D imaging: acquiring line by line

Typical Gradient Echo sequence



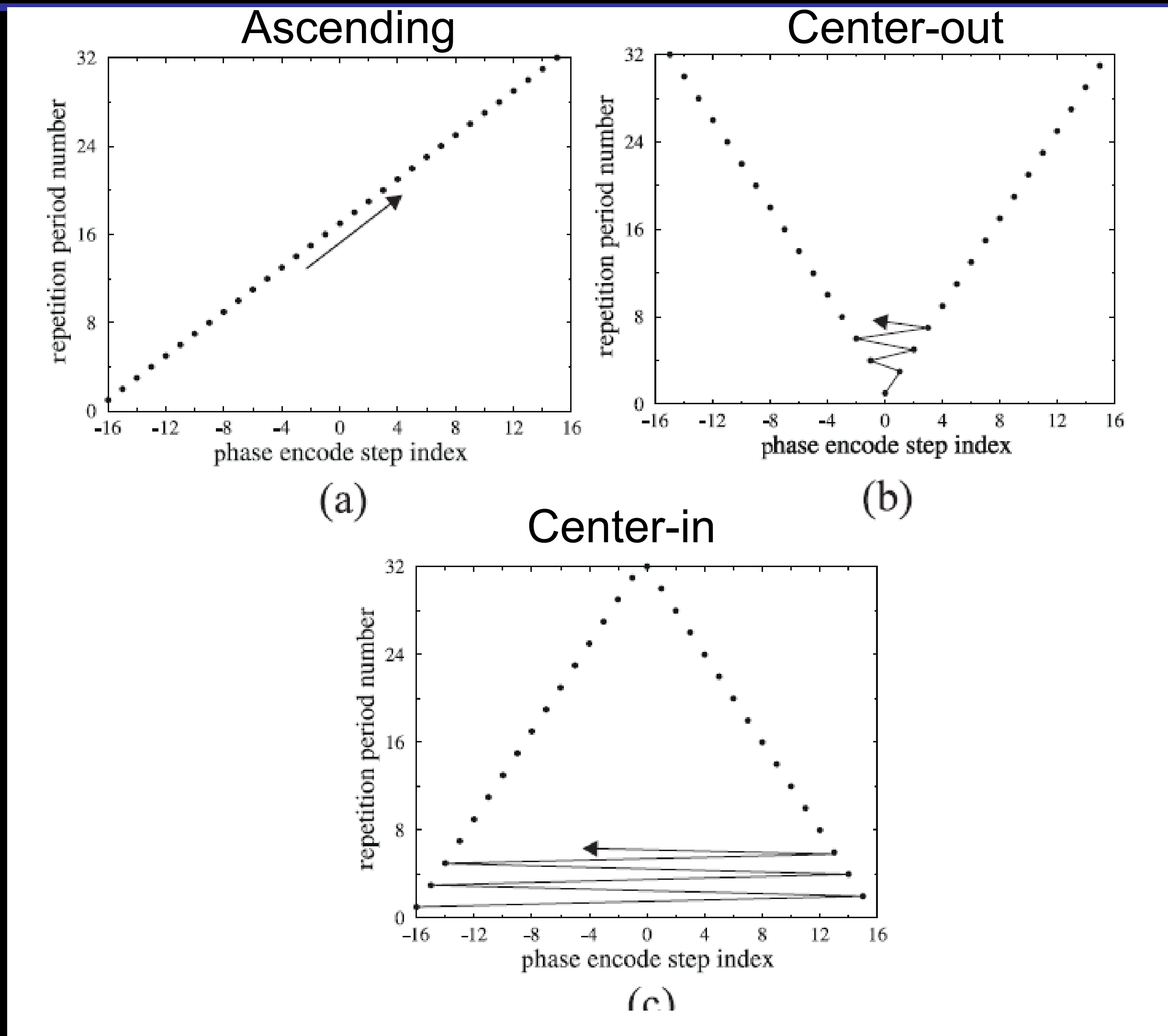
k-space

Image space

Total imaging time for a single slice – N_y x TR

Sampling of the signal line by line forming the image in Spatial frequency domain

Phase encoding order (Cartesian)

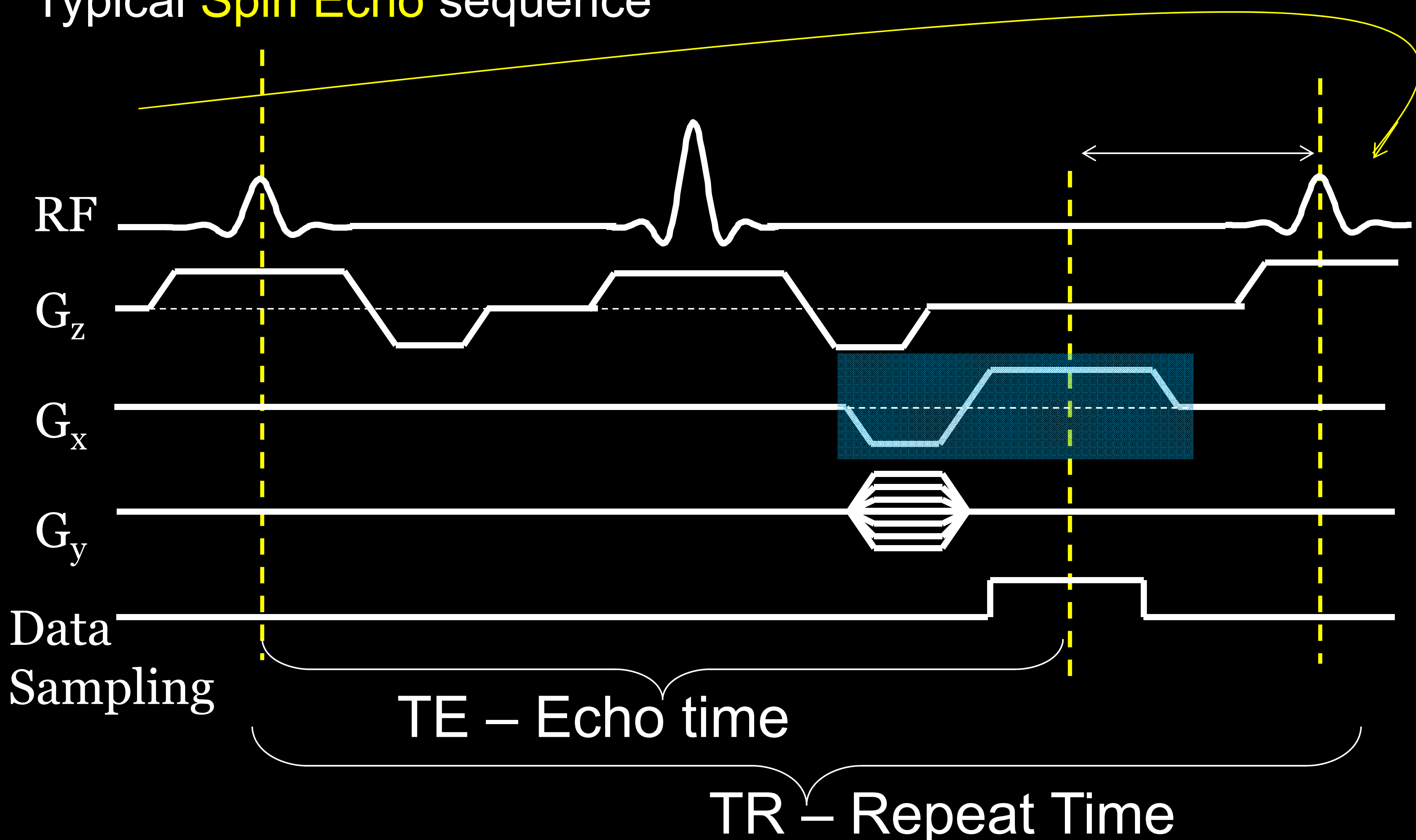


Phase Encoding vs. Frequency Encoding

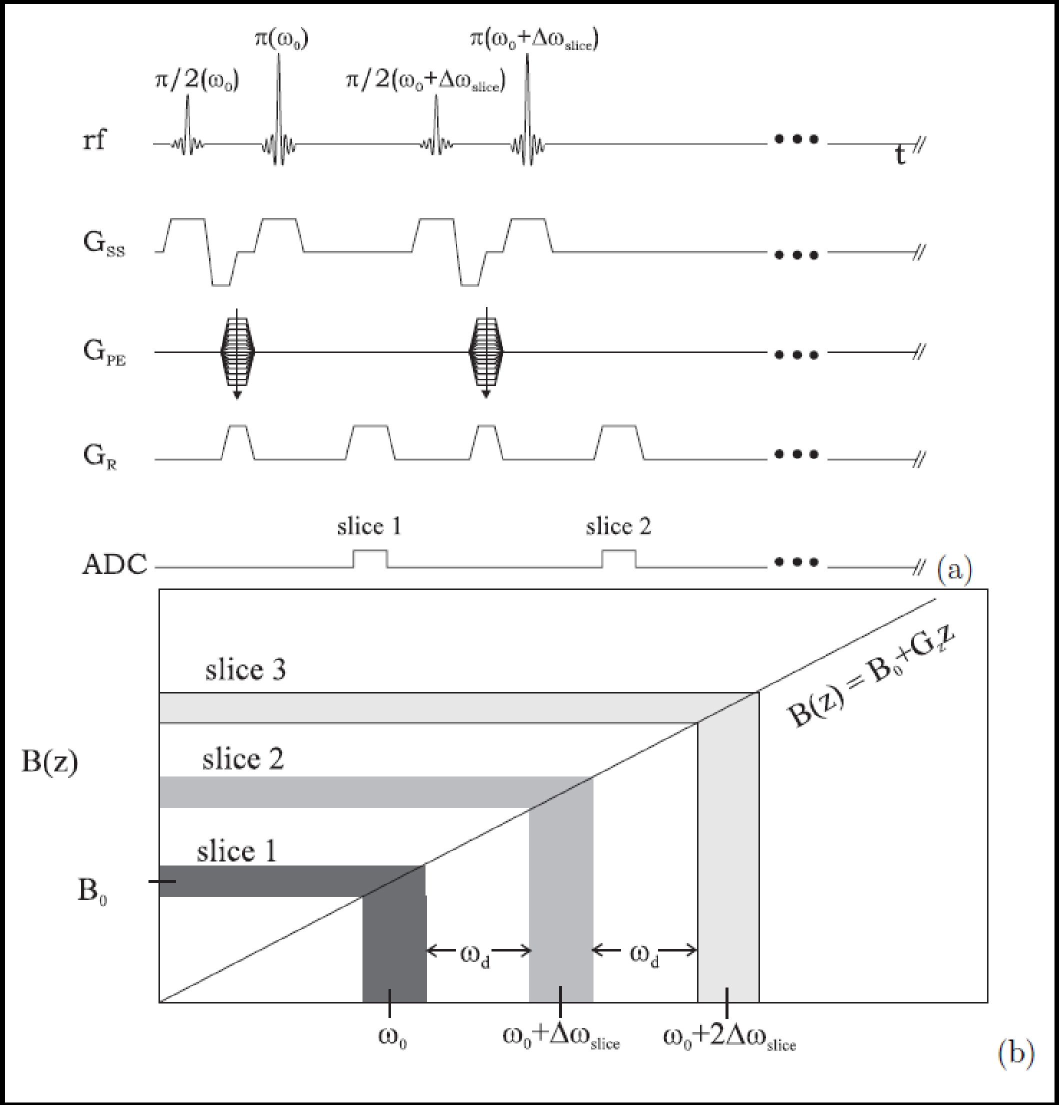
- Acquisition: Discrete vs continuous accrual of phase
- Time-distance between every adjacent points along phase vs frequency encoding is different (Δt vs TR)
- Difference in time between position encoding and data acquisition
- Spatial/k direction

Acquiring the data… line by line

Typical Spin Echo sequence

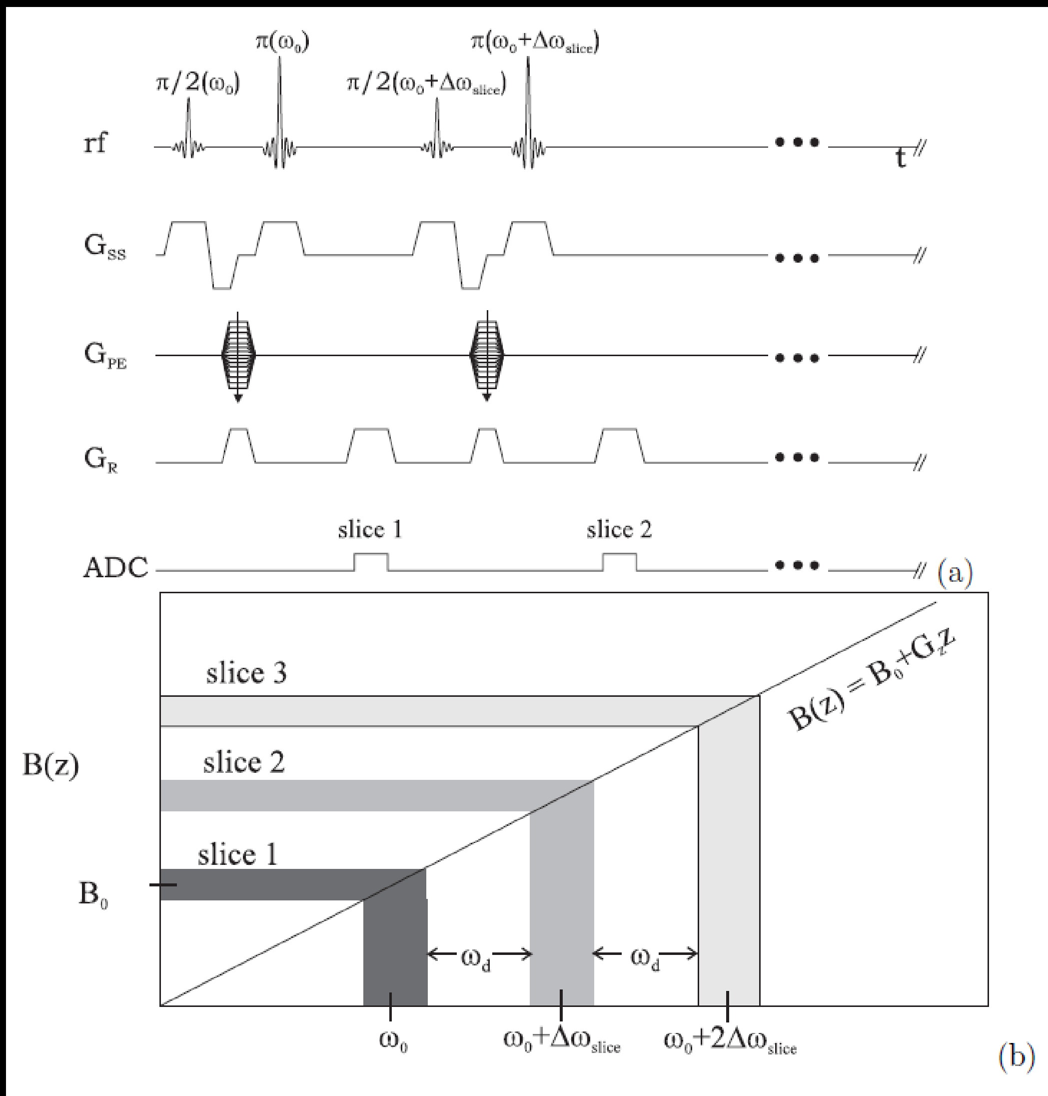


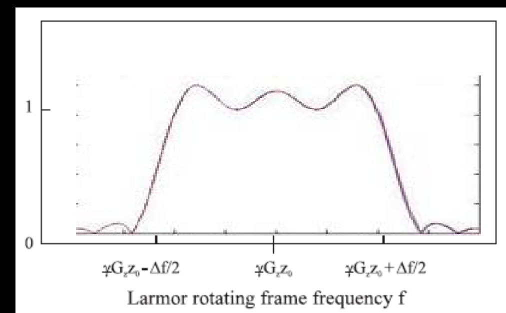
Multi-slice 2D imaging

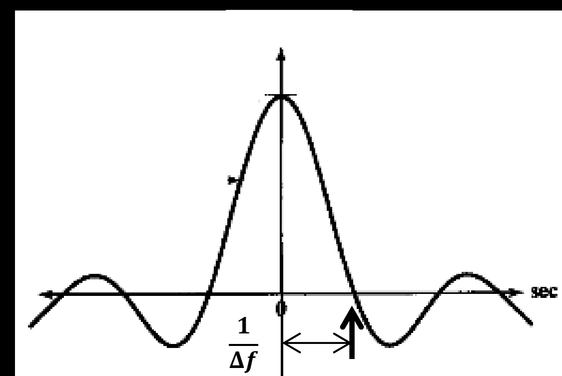


- **Using the dead time to acquire data from multiple slices within the same TR**
- **Same G_{ss} but varying RF center frequency**
- **Slice gap is usually needed due to imperfect RF frequency profiles**

Multi-slice 2D imaging





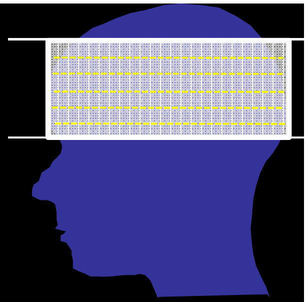


$$\gamma G_{ss}(TH + d) = \Delta\omega_{slice}$$

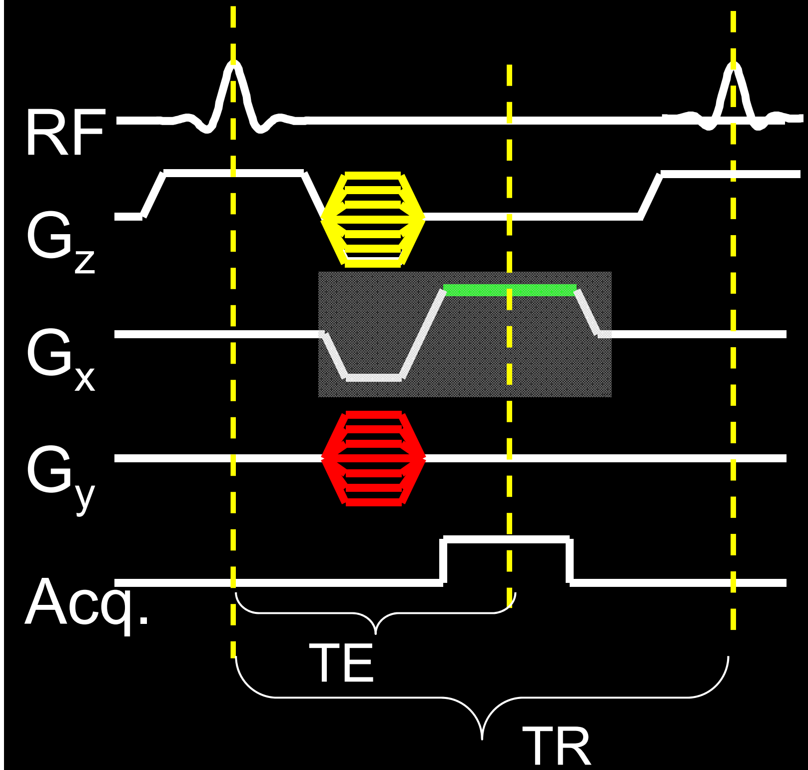
$$d = \frac{\left(\Delta\omega_{slice} - 2\pi BW_{rf}\right)}{\gamma G_s}$$

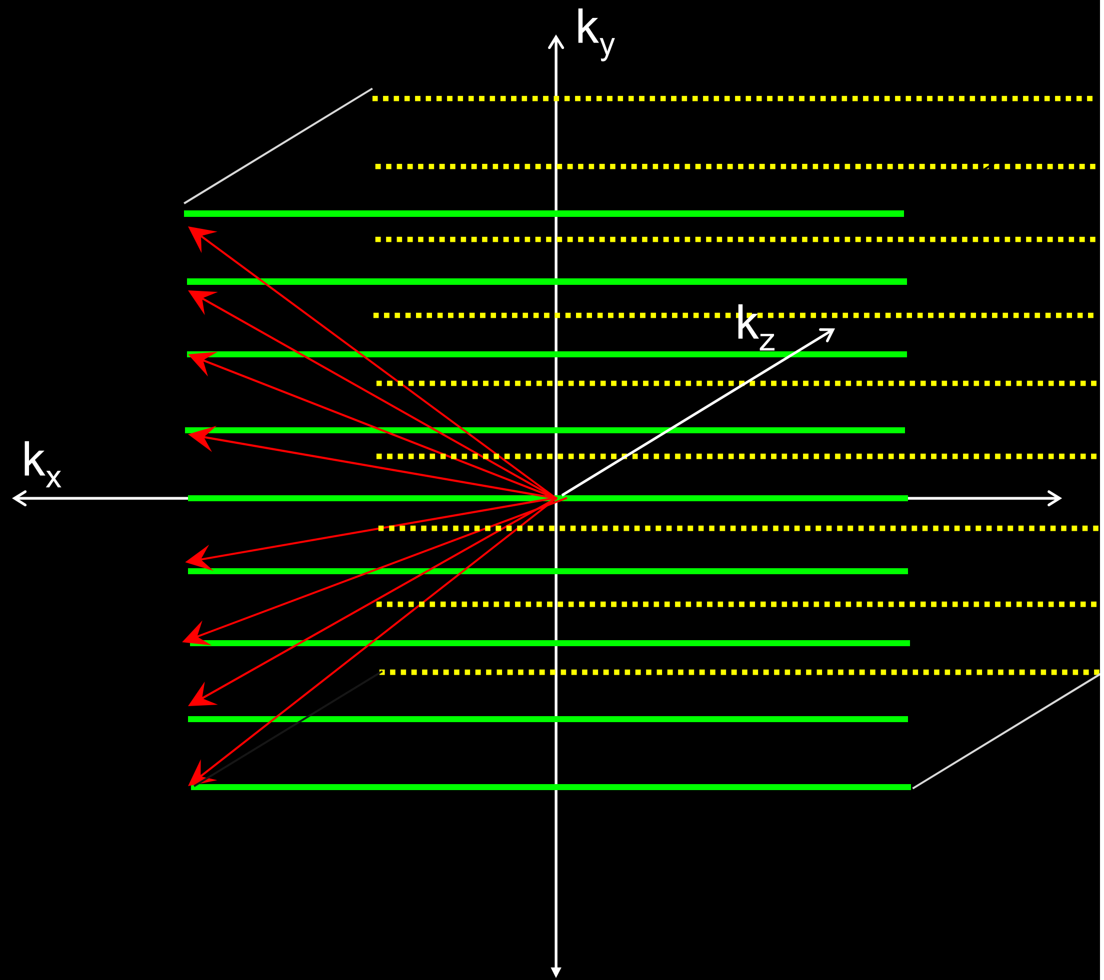
Slice interleaved acquisition - Odd/Even

3D volumetric imaging



3D GRE - with slab excitation and partition encoding

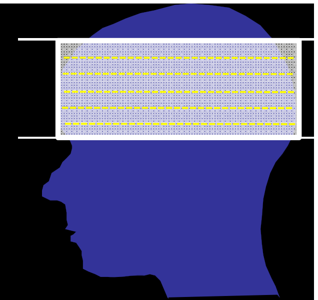




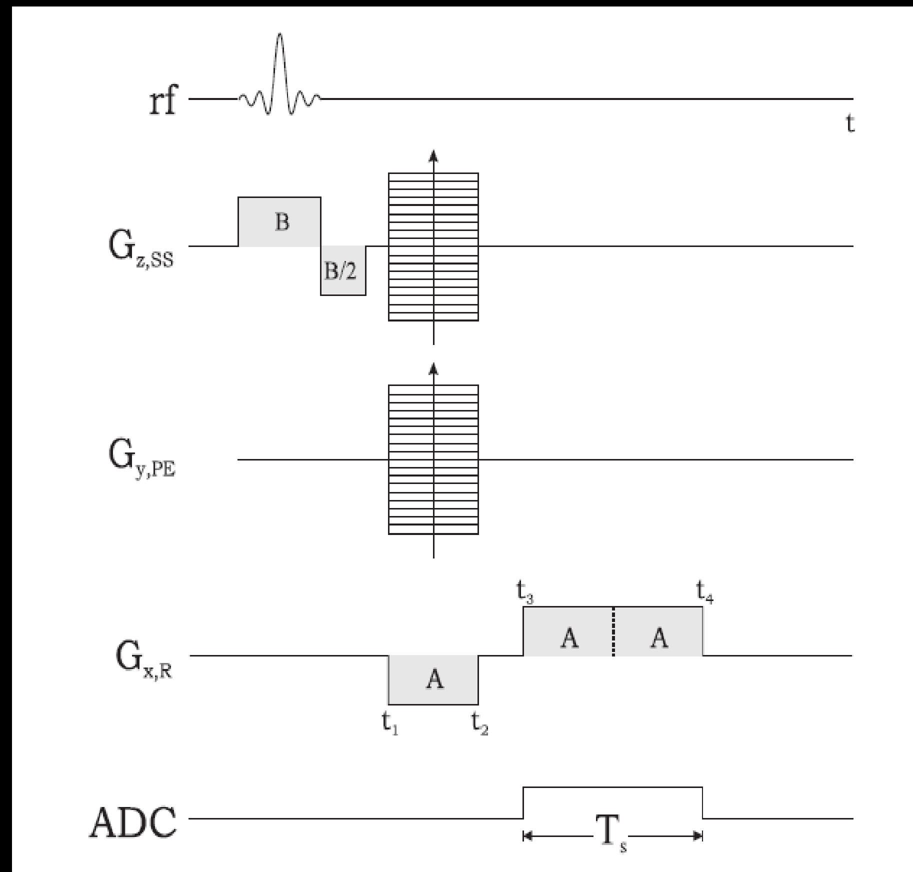
Total Imaging time – $N_z \times N_y \times TR$

Slab Excitation and partition encoding in the z direction.

3D imaging



- Additional PE along slice selection
- Volumetric excitation (slab instead of slice)
- 3D iFFT
- Pros (vs. multi-slice 2D):
  - High resolution on SS dimension
  - High SNR
- Cons (vs. multi-slice 2D):
  - Longer scan time
  - Min slice# per slab for FFT
  - Sensitive to motion



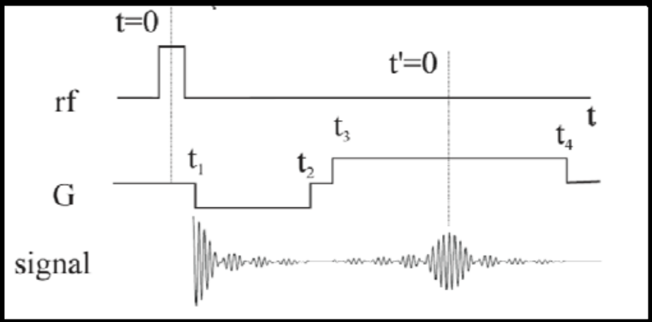
Homework

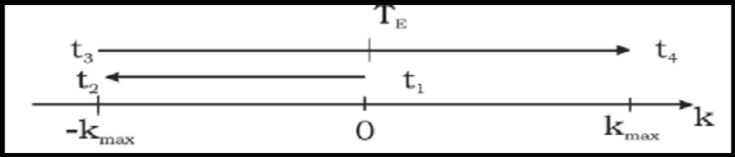
- Probs. 10.1 - 10.4, 10.7

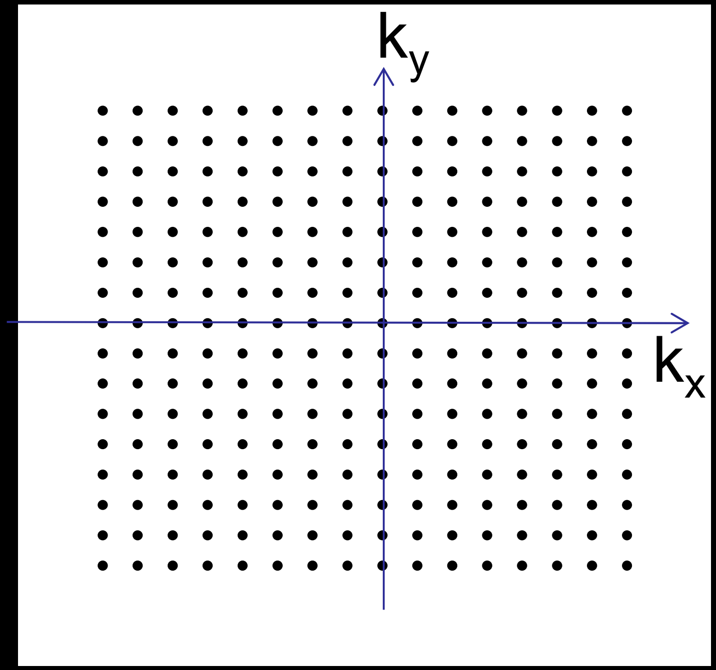
Next Session

Chapter 11 and 12.1-12.2

Multidimensional Spatial Encoding (2D/3D imaging)







$$S(k_x) = \int dx \cdot \rho(x) \cdot e^{-i \cdot \phi_x}$$

$$\phi_x = \gamma \cdot G_x \cdot \Delta t \cdot x = -2\pi \cdot \Delta k_x \cdot x$$