This class - Key points

• Noise (signal to noise ratio) -> relation to experimental parameters
• SNR in magnitude and phase domain
• Contrast, its optimization
• Partial voluming
• Affect of common exogenous contrast agents
Noise

Any physical measurement using an instrument/sensor has noise and it is our ability to detect the signal above the noise that characterizes a successful measurement.

A measure of a successful experiment is the ratio

\[
\text{SNR} = \frac{S}{\sigma_0}
\]
Intro

• Noise is practically inevitable
  – Random / systematic
  – Physiological

• Signal-to-noise ratio (SNR) determines the effectiveness of the imaging

• SNR affects CNR, which determines the usefulness of the image

• Good CNR requires good SNR, but good SNR not necessary means good CNR
Signal and noise

• Consider equilibrium magnetization and receive coil field

\[
s \equiv \hat{\rho}_m \propto \omega_0 M_0 B_\perp V_{\text{voxel}} \\
\propto \gamma B_0 \rho_0 \frac{\gamma^2 h^2}{4kT} B_0 B_\perp V_{\text{voxel}} \\
\propto \frac{B_0^2 V_{\text{voxel}}}{T} \rho_0 B_\perp
\]

• Many factors being ignored or constant here
  – T1, T2, T2*
  – RF field: flip angle, B1 field homogeneity
  – Sequence type, TE/TR/TI/…
Signal and noise

- Revisiting the measured signal

1D:

\[
\hat{\rho}_m(q\Delta x) = \frac{1}{N} \sum_{p=-\frac{N}{2}}^{\frac{N}{2}-1} s(p\Delta k)e^{i2\pi pq/N} \\
= \Delta x \sum_{q=-\frac{N}{2}}^{\frac{N}{2}-1} \hat{\rho}(q\Delta x) \delta(x - q\Delta x)
\]

3D:

\[
\hat{\rho}_m(p\Delta x, q\Delta y, r\Delta z) = \frac{1}{N_x N_y N_z} \sum_{p',q',r'} s(p'\Delta k_x, q'\Delta k_y, r'\Delta k_z)e^{i2\pi \left(\frac{pp'}{N_x} + \frac{qq'}{N_y} + \frac{rr'}{N_z}\right)} \\
= \Delta x \Delta y \Delta z \sum_{p,q,r} \hat{\rho}(p\Delta x, q\Delta y, r\Delta z) \delta(x - p\Delta x) \delta(y - q\Delta y) \delta(z - r\Delta z)
\]
Signal and noise

- **Noise in MRI**
  - Thermal noise (body/object, coil, electronics)
  - Systematic noise (scanner, AC power, etc)
  - Physiological noise (heartbeat, respiration, etc)

- **Thermal noise estimation**
  \[
  \sigma_{\text{thermal}}^2 = \sigma_{\text{body}}^2 + \sigma_{\text{coil}}^2 + \sigma_{\text{electronics}}^2
  \]
  \[
  \propto 4kT \cdot R \cdot BW_{\text{coil}}
  \]
  \[
  R_{\text{eff}} = R_{\text{body}} + R_{\text{coil}} + R_{\text{electronics}}
  \]
  \[
  BW_{\text{coil}} \equiv BW_{\text{readout}} : \text{bandwidth of reception}
  \]

*Note that this is noise in k-space domain – What is relevant for us is the noise in image domain*
\( S_m(k) = s(k) + \epsilon(k) \)

- For thermal (or white/Gaussian) noise

\[
R_\epsilon(\tau) \equiv \epsilon(k_p)\epsilon^*(k_q) \big|_{\tau=(k_p-k_q)} = \sigma_m^2 \delta(\tau)
\]

Spectral density: \( r_\eta(f) \equiv \mathcal{F}[R_\epsilon(\tau)] = \sigma_m^2 \)

Fourier Transform of noise: \( \mathcal{F}[\epsilon(k)] = \eta(x) \)
Noise vs. Imaging parameters

• **Noise in MR image**

\[
\eta(p\Delta x) = iDFT[\varepsilon(k)] = \frac{1}{N} \sum_{p'} \varepsilon(p'\Delta k)e^{i2\pi p'\Delta k p \Delta x}
\]

• **Properties of noise in image**

\[
\text{mean}(\eta(p\Delta x)) = iDFT[\varepsilon(k)] = 0 \\
\text{var}(\eta(p\Delta x)) = DFT[R_\varepsilon(\tau)] = \frac{\sigma_m^2}{N}
\]

• **Implications**
  - White noise in k-space results in white noise in image
  - Noise variance (std) in image is \(N(\sqrt{N})\) times smaller than in k-space
Improving SNR

• Averaging over multiple repeated acquisitions

\[ s_{m,\text{ave}}(k) = \frac{1}{N_{\text{acq}}} \sum_i s_{m,i}(k) = s_m(k) \]

\[ \sigma_{m,\text{ave}} = \sqrt{\frac{\sum_i \sigma_{m,i}^2}{N_{\text{acq}}}} = \frac{\sigma_m}{\sqrt{N_{\text{acq}}}} \]

Thus

\[ SNR_{m,\text{ave}} = \sqrt{N_{\text{acq}}} SNR_m \]

• Properties
  – True for both k-space and images
  – Valid only when noise are uncorrelated between acquisitions, systematic noise such as artifacts will not be reduced
Improving SNR: Multiple averages

Improving SNR

- Averaging can be done over images or k-space
- Two modes for averaging over $k_y$

$$\sigma_{0,avg} = \frac{\sigma_0}{\sqrt{N_{acq}}}$$

$$\text{SNR}_{ave} = \sqrt{N_{acq}} \cdot \text{SNR}$$
Improving SNR

- Use multiple smaller surface coils (phased array coils)
  - Smaller coil -> less coupling -> less noise
  - Rapidly weakening receive fields $B$ with distance
  - Sum-of-square offers optimal results over linear combination

http://www.insightmri.com/
Measuring SNR

- Mean and std of ROI in homogeneous regions
- Use std in background ROI to estimate $\sigma_0$ ($\sigma_{bg} = 0.655 \sigma_0$, Rayleigh distribution)

$\sigma_{oj} > \sigma_{bg}$

Gaussian vs. Rayleigh;

- Scan twice, add and subtract the two images to get mean and std ($\sigma_{sub} = \sqrt{2}\sigma_0$)
- Collect multiple volumes of images for voxel wise SNR (tSNR)
Measuring SNR

\( \sigma_{bg} = 0.655 \sigma_0, \text{ Rayleigh distribution} \)

\[ \sigma_{oj} > \sigma_{bg} \]

Single Acq

subtracted
SNR dependence on imaging parameters

Voxel signal $\propto \Delta x$ (Voxel volume)

Noise standard deviation $\sigma \propto \frac{1}{\sqrt{N_x}}$

Noise standard deviation $\sigma \propto \frac{1}{\sqrt{N_{acq}}}$

Noise standard deviation $\sigma \propto \sqrt{BW_{readout}}$

$$SNR/voxel \equiv \frac{Signal_{voxel}}{\sigma} \propto \frac{\Delta x \Delta y \Delta z \sqrt{N_{acq}}}{\sqrt{BW_{read} \sqrt{N_x N_y N_z}}}$$
SNR/voxel \equiv \frac{\text{Signal}_{\text{voxel}}}{\sigma} \propto \frac{\Delta x \Delta y \Delta z \sqrt{N_{\text{acq}}}}{\sqrt{\frac{BW_{\text{read}}}{N_x N_y N_z}}}

\frac{1}{\Delta t} = \gamma \cdot G_x \cdot N_x \cdot \Delta x \text{ Total read bandwidth}

\frac{\text{SNR}}{\text{voxel}} \propto \Delta x \Delta y \Delta z \sqrt{N_{\text{acq}} N_x N_y N_z \Delta t}

T_s = N_x \Delta t

\frac{\text{SNR}}{\text{voxel}} \propto \Delta x \Delta y \Delta z \sqrt{N_{\text{acq}} N_y N_z T_s}
\[ \text{SNR/voxel} \propto \frac{\Delta x \Delta y \Delta z \sqrt{N_{acq}}}{\sqrt{\frac{\text{BW}_{\text{read}}}{N_x N_y N_z}}} \]

\[ \text{SNR/voxel} \propto \Delta x \Delta y \Delta z \sqrt{N_{acq} N_x N_y N_z \Delta t} \]

\[ \text{SNR/voxel} \propto \Delta x \Delta y \Delta z \sqrt{N_{acq} N_y N_z T_s} \]

(a) \[ L_x = N_x \Delta x \]
(b) \[ L_y = N_y \Delta y \]
(c) \[ L_z = N_z \Delta z \]
(d) \[ T_s = N_x \Delta t \]
(e) \[ \text{BW}_{\text{read}} = \frac{1}{\Delta t} = \gamma G_x L_x \]
(f) \[ \text{BW/voxel} = \frac{\text{BW}_{\text{read}}}{N_x} \]

<table>
<thead>
<tr>
<th>Case</th>
<th>( \Delta x )</th>
<th>( N_x )</th>
<th>( L_x )</th>
<th>( G_x )</th>
<th>( \Delta t )</th>
<th>( T_s )</th>
<th>SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \Delta x_0 )</td>
<td>( N_0 )</td>
<td>( L_0 )</td>
<td>( G_0 )</td>
<td>( \Delta t_0 )</td>
<td>( T_{s,0} )</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>( \Delta x_0 / 2 )</td>
<td>( N_0 )</td>
<td>( L_0 / 2 )</td>
<td>( G_0 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>( \Delta x_0 / 2 )</td>
<td>( N_0 )</td>
<td>( L_0 / 2 )</td>
<td>( 2G_0 )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
\[
\text{SNR/voxel} \propto \frac{\Delta x \Delta y \Delta z \sqrt{N_{\text{acq}}}}{\sqrt{\frac{BW_{\text{read}}}{N_x N_y N_z}}} \\
\text{SNR/voxel} \propto \Delta x \Delta y \Delta z \sqrt{N_{\text{acq}} N_y N_z \Delta t} \\
\text{SNR/voxel} \propto \Delta x \Delta y \Delta z \sqrt{N_{\text{acq}} N_y N_z T_s}
\]

\[(a) \quad L_x = N_x \Delta x \quad (b) \quad L_y = N_y \Delta y \quad (c) \quad L_z = N_z \Delta z \quad (d) \quad T_s = N_x \Delta t \quad (e) \quad BW_{\text{read}} = \frac{1}{\Delta t} = \gamma G_x L_x \quad (f) \quad BW/\text{voxel} = BW_{\text{read}}/N_x
\]

<table>
<thead>
<tr>
<th>Case</th>
<th>(\Delta x)</th>
<th>(N_x)</th>
<th>(L_x)</th>
<th>(G_x)</th>
<th>(\Delta t)</th>
<th>(T_s)</th>
<th>SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(\Delta x_0)</td>
<td>(N_0)</td>
<td>(L_0)</td>
<td>(G_0)</td>
<td>(\Delta t_0)</td>
<td>(T_{s,0})</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>(\Delta x_0/2)</td>
<td>(N_0)</td>
<td>(L_0/2)</td>
<td>(G_0)</td>
<td>(2\Delta t_0)</td>
<td>(2T_{s,0})</td>
<td>(1/\sqrt{2})</td>
</tr>
<tr>
<td>7</td>
<td>(\Delta x_0/2)</td>
<td>(N_0)</td>
<td>(L_0/2)</td>
<td>(2G_0)</td>
<td>(\Delta t_0)</td>
<td>(T_{s,0})</td>
<td>(1/2)</td>
</tr>
</tbody>
</table>
SNR dependence on imaging parameters

\[ \text{SNR/voxel} \equiv \Delta y \Delta z \sqrt{N_y N_z} \]

\[ \frac{\text{SNR}}{\text{voxel}} \propto \Delta x \Delta y \Delta z \sqrt{N_{acq} N_y N_z T_s} \]

2D vs 3D

\[ \Delta z \equiv TH \]

\[ (\text{SNR/voxel})|_{2D} \propto \Delta x \Delta y TH \sqrt{N_y T_s} \]
SNR vs. $B_0$

- $B_0$ affects MRI signal and noise in a number of ways, e.g. $M_0$, T1, T2, field inhomogeneity, and thus T2*

  \[ S \propto \frac{B_0^2 V_{\text{voxel}}}{T} \propto \omega_0^2 \]

  \[ R_{\text{eff}}(\omega_0) \approx \begin{cases} 
  R_{\text{coil}}(\omega_0) + R_{\text{electronics}}(\omega_0) & \omega_0 \ll \omega_{0,\text{mid}} \\
  R_{\text{sample}}(\omega_0) & \omega_0 \gg \omega_{0,\text{mid}}
  \end{cases} \]

  \[ \text{SNR}(\omega_0) \propto \begin{cases} 
  \omega_0^{7/4} & \omega_0 \ll \omega_{0,\text{mid}} \\
  \omega_0 & \omega_0 \gg \omega_{0,\text{mid}}
  \end{cases} \]

- Higher $B_0 \rightarrow$ longer T1, shorter T2/T2*

\[ \sigma_{\text{thermal}} \propto \begin{cases} 
  1/4 & \omega_0 \ll \omega_{0,\text{mid}} \\
  1 & \omega_0 \gg \omega_{0,\text{mid}}
  \end{cases} \]
SNR dependence on imaging parameters

\[ \text{SNR/voxel} \propto \frac{\Delta x \Delta y \Delta z \sqrt{N_{\text{acq}}}}{\sqrt{\frac{\text{BW}_{\text{read}}}{N_x N_y N_z}}} \]

Proportional
Inv. Proportional

Total imaging time \( T_T \)

User input para
Implicit para
Outcomes
Imaging efficiency

- Total imaging time
  \[ T_T = N_{acq} N_y N_z T_R \]

- Imaging efficiency (with otherwise fixed parameters)
  \[ \gamma \equiv \frac{SNR/voxel}{\sqrt{T_T}} \propto \Delta x \Delta y \Delta z \sqrt{T_s} \]

- Implications
  - Large voxels yield better SNR efficiency
  - So does longer readout (or lower RO bandwidth). But sometimes T2* decay may negate the effects
Contrast, CNR and visibility

- High SNR is only half the story

- The other half is to be able to distinguish different object/structure/properties, i.e. have enough contrast with the presence of noise
Contrast, CNR and visibility

• Contrast: simply the signal difference
  \[ C_{AB} \equiv S_A - S_B \]

• CNR: contrast-to-noise ratio
  \[ CNR_{AB} \equiv SNR_A - SNR_B \]

when \( \sigma_A = \sigma_B = \sigma_0 \),

\[ CNR_{AB} = \frac{C_{AB}}{\sigma_0} \]
Contrast, CNR and visibility

- **Visibility**

\[ \nu_{AB} \equiv \frac{C_{AB}}{\sigma_{eff}} = \frac{C_{AB}}{\sigma_0} \sqrt{N_{acq}} = \frac{C_{AB}}{\sigma_0} \sqrt{n_{voxel}} \]

- **The Rose Criterion**

\[ CNR_{AB} > 3 \sim 5 \]
Contrast, CNR and visibility

SNR=$\infty$

SNR=4

SNR=2

SNR=1

Higher signal

Larger size

(a) (b)

(c) (d)
Homework

- Prob 15.1, 15.3, 15.4, 15.8, 15.10, 15.12, 15.13

Next Session

Chapter 15.3-15.6
Contrast mechanisms in MRI

- MRI is a multiple contrast imaging modality
- Or multiple methods for the same contrast
  - Example: T1W: short TE/TR GE, or IR
  
  Signal nulling: IR, selective excitation/saturation, dephasing

Intrinsic:
- spin density
- T1/T2/T2*/T1ρ
- Diffusion
- Chemical shift
- Susceptibility
- Temperature
- ...

Physiological:
- Blood flow
- Perfusion
- Blood oxygenation
- Tissue elasticity
- ...

Sequence specific:
- Steady state (chap.18)
- Selective excitation/suppression
- ...

- Or multiple methods for the same contrast
Basic Contrast Mechanisms: Relaxation effects

\[
\frac{d\vec{M}}{dt} = \gamma \cdot \vec{M} \times B + \frac{M_0 - M_z}{T_1} - \frac{M_{xy}}{T_2}
\]

- $M_0$ is the equilibrium magnetization
- $M_{xy}$ is the transverse magnetization
## Contrast

<table>
<thead>
<tr>
<th>Tissue</th>
<th>$T_1$ (ms)</th>
<th>$T_2$ (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>gray matter (GM)</td>
<td>950</td>
<td>100</td>
</tr>
<tr>
<td>white matter (WM)</td>
<td>600</td>
<td>80</td>
</tr>
<tr>
<td>muscle</td>
<td>900</td>
<td>50</td>
</tr>
<tr>
<td>cerebrospinal fluid (CSF)</td>
<td>4500</td>
<td>2200</td>
</tr>
<tr>
<td>fat</td>
<td>250</td>
<td>60</td>
</tr>
<tr>
<td>blood$^3$</td>
<td>1200</td>
<td>100-200$^4$</td>
</tr>
</tbody>
</table>
Once you tip the magnetization of a tissue, it takes a finite amount of time for it to recover.

With repeated rf pulse excitations during image acquisition, the time intervals of repetition and acquisition (TR,TE), along with T1 and T2 values of different tissue determine the final contrast.
Magnetization equations

\[ M_0 \quad M_0(1-e^{-T/T_1}) \quad M_0(1-e^{-(t-T_2)/T_2}) \text{ etc.} \]

\[ M_\Delta(t) \quad M_0 e^{-T/T_1} \quad M_0(1-e^{-T/T_1})e^{-(t-T_2)/T_2} \text{ etc.} \]
Magnetization equations

\[ M_z(t) = M_z(0) \cdot e^{-\frac{t}{T_1}} + M_0 \cdot \left(1 - e^{-\frac{t}{T_1}}\right) \]

\[ M_{xy}(t) = M_{xy}(0) \cdot e^{-\frac{t}{T_2}} \]

\[ M_z(0) = M_0 \cdot \cos(\theta) \]

\[ M_{xy}(0) = M_0 \cdot \sin(\theta) \]

\[ M_z(TR^-) = M_0 \cdot \left(1 - e^{-\frac{TR}{T_1}}\right) \]

\[ M_{xy}(TE) = M_0 \cdot \left(1 - e^{-\frac{TR}{T_1}}\right) \cdot e^{-\frac{TE}{T_2}} \]

\[ TR \gg T_2 \]
Magnetization equations

Take-aways from the math

Steady state reached by the second rf pulse

Steady state meaning... $M_z(\text{TR})$ and $M_{xy}(\text{TE})$ are constant
Contrast

Longitudinal Magnetization Recovery

GM

WM

CSF

time t
Different tissues have different amounts of water and hence different densities of $^1$H protons within them.
Contrast

Longitudinal Magnetization Recovery

$M_{xy}(t) = M_{xy}(0) \cdot e^{-\frac{TE}{T_2}}$

Echo Time, TE

GM

WM

CSF

time $t$

TR

$\tau$
Contrast...\(\rho_0\) weighting

Longitudinal Magnetization Recovery

- GM (Gray Matter)
- WM (White Matter)
- CSF (Cerebrospinal Fluid)

TR (Repetition Time)

Echo Time, TE
Contrast... $\rho_0$ weighting

Long repetition time - TR
Short echo time - TE

Longitudinal Magnetization Recovery

TR

GM

WM

CSF

Echo Time, TE

time t
Contrast

Longitudinal Magnetization Recovery

TR

TE

time t

GM

WM

CSF

RF

q_x

q_y

q_z

Acq
Contrast... $T_1$ weighting

Short repetition time - TR
Short echo time - TE
Contrast

Longitudinal Magnetization Recovery

TR

CSF

WM

GM

TE
Contrast... $T_2$ weighting

Long repeat time - TR
Long Echo time - TE

TR

Longitudinal Magnetization Recovery

CSF
WM
GM

time t

TE
Most important contrast types

\( \rho_0, \text{T1 and T2 (T2*)} \)

\[
S_A(\text{TR,TE}) = \rho_{0,A}(1 - e^{-\text{TR}/T_{1,A}})e^{-\text{TE}/T_{2,A}}
\]

\[
S_B(\text{TR,TE}) = \rho_{0,B}(1 - e^{-\text{TR}/T_{1,B}})e^{-\text{TE}/T_{2,B}}
\]

\[
C_{AB}(\text{TR,TE}) = S_A(\text{TR,TE}) - S_B(\text{TR,TE})
\]
Most important contrast types

- **Spin density weighting**

\[ S(TR,TE) = \rho_0 (1 - e^{-TR/T_1}) e^{-TE/T_2^*} \]

\[ TR \rightarrow T_1 \quad \Rightarrow \quad e^{-TR/T_1} \rightarrow 0 \]

\[ TE \rightarrow T_2^* \quad \Rightarrow \quad e^{-TE/T_2^*} \rightarrow 1 \]

\[ C_{AB}(TR,TE) \approx \rho_{0,A} - \rho_{0,B} \]

Note: \( \rho_{0,A} \) and \( \rho_{0,B} \) are effective spin density
Most important contrast types

- **T1 weighting**

\[ C_{AB}(TR) \approx \rho_{0,A} - \rho_{0,B} - \left( \rho_{0,A}e^{\frac{TR}{T_{1,A}}} - \rho_{0,B}e^{\frac{TR}{T_{1,B}}} \right) \]

\[ TR_{opt} = \frac{\ln\left(\frac{\rho_{0,B}}{T_{1,B}}\right) - \ln\left(\frac{\rho_{0,A}}{T_{1,A}}\right)}{\frac{1}{T_{1,B}} - \frac{1}{T_{1,A}}} \]

\[ = T_{1,A} |_{T_{1,A}} \]

T1 values at 3T
- GM: ~1200ms
- WM: ~800ms
- CSF: ~2-4s
Most important contrast types

\[ S(\text{TR}, \text{TE}) = \rho_0 (1 - e^{-\text{TR}/\text{T}_1}) e^{-\text{TE}/\text{T}_2^*} \]

- **T2\(^*\) weighting**

  use  \( \text{TR} \ll \text{T}_1 \)  \( \Rightarrow \)

  \[ e^{-\text{TR}/\text{T}_1} \rightarrow 0 \]

\[
C_{AB}(\text{TE}) \bigg|_{\text{TE}_{\text{opt}}} = \rho_{0,A} e^{\ln \left( \frac{\rho_{0,B}}{\text{T}_2,B} \right)^*} \ln \left( \frac{\rho_{0,A}}{\text{T}_2,A} \right)^* \frac{\text{TR}}{\text{T}_2,B} \]

\[
= T_{2,A}^* \left| T_{2,A}^* \approx T_{2,B}^* \right.
\]
T1 weighting with inversion recovery (Chap 8.4)

- IR may double the T1 contrast relative to GE
- Optimal TI (?)
- Potential of nulling certain tissues
IR examples

- T1W
- CSF nulling, T2W
- WM+CSF nulling
- GM+CSF nulling

- T2W
- Fat nulling, T2W
Contrast enhancement with T1-shortening agents (Gd)

- With T1 shortened, target tissue will have increased signal in short TR GE images

- T1 shortening effect proportional to agent concentration

- Target tissue example:
  - Blood (MR angiography, perfusion weighted imaging)
  - Tumor
  - Vessel wall lesion
Contrast enhanced images

Pre-CA

Post-CA

(a)

(b)

Graph showing signal intensity over time (s) with lines for benign tumor, normal breast tissue, and fat.
CNR, partial volume and resolution

\[ \mathcal{N}_{AB} \equiv \frac{C_{AB}}{\sigma_{eff}} = \frac{C_{AB}}{\sigma_0} \sqrt{N_{acq}} = \frac{C_{AB}}{\sigma_0} \sqrt{n_{voxel}} \]

- Resolution \( \sigma_0 \) -> \( n_{voxel} \) -> \( \mathcal{N}_{AB}(?) \)

- Partial volumed voxel contains >1 tissues/objects

- Partial volume effect (PVE) is inevitable in in-vivo MR imaging
  - Finite voxel size (0.25mm ~ 4mm)
  - Complex tissue boundaries
  - Micro vasculature/structures
  - ...
CNR, partial volume and resolution

- **Size of A:** \( \alpha \Delta x \), \( \alpha < 1 \)
- **Size of B:** \( \Delta x \) or \( \beta \Delta x \), \( \beta < 1 \)
- **In both scenarios:** \( C_{AB} = \alpha (S_A - S_B) \)
- **Noise \( \sigma \):**
  \[ \sigma(\alpha) = \sigma_0, \quad (\alpha L_x, N) \]
  \[ \sigma(\alpha) = \sigma_0 / \sqrt{\alpha}, \quad (L_x, \alpha N) \]
How PVE affects the image:

• Blurring
• Signal dephase ($T2^'/T2^*$, BOLD imaging, chap 25)
• Double/multiple $T2/T2^*$ exponential decay
• Signal cancellation (water/fat)

Note the similarity with Fig. 17.8
Courtesy of Manju
Signal and noise of phase images

- $S_{phase}$ is affected by chemical shift, local susceptibility and/or motion, offering different contrast than magnitude images.

- $S_{phase}$ is a relative value since baseline $\phi_0$ is unknown.

- Only the contrast in phase has useful information.

- $\sigma_{phase} = 1/SNR_{mag}$

- Phase may still provides sufficient contrast when magnitude fails.
Phase imaging applications

- Chemical shift (fat/water) imaging (Chap. 17)
- $B_0$ field homogeneity mapping (Chap. 20)
- Phase contrast MR angiography (Chap. 24)
- Blood flow quantification (Chap. 24)
- Susceptibility Weighted Imaging (SWI, Chap. 25)
- ...
Homework

- Prob 15.1, 15.3, 15.4, 15.8, 15.10, 15.12, 15.13

Next Session

Chapter 15.3-15.6
Most important contrast types

- Examples:
  - T1 values at 3T
    - GM: ~1200ms
    - WM: ~800ms
    - CSF: ~2-4s