Chapter 15 – Signal, Contrast and Noise

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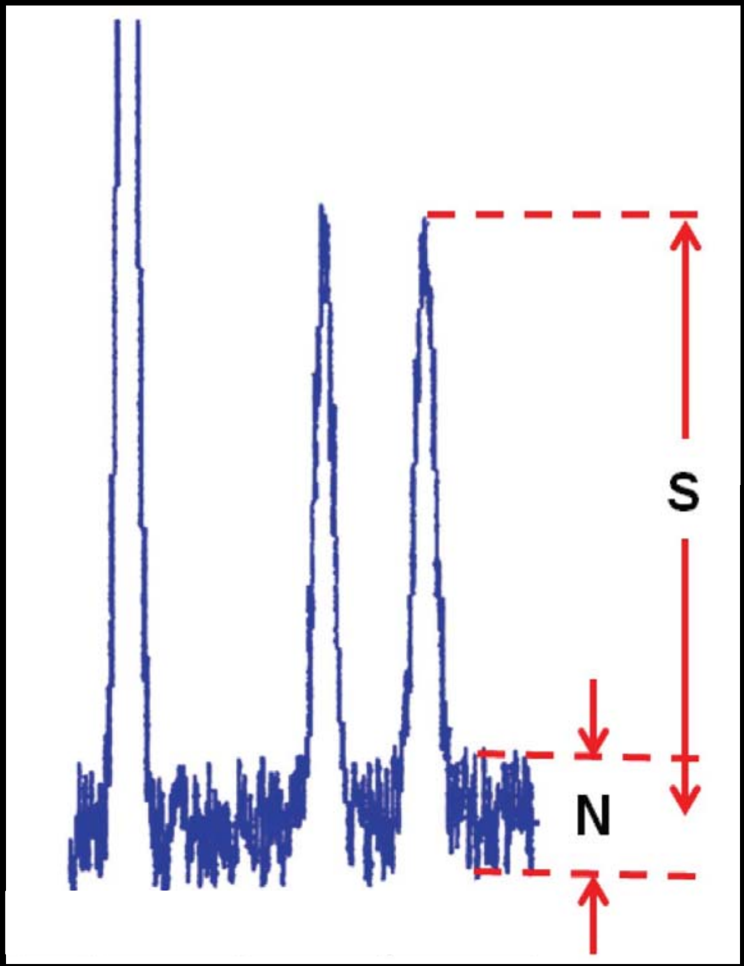
This class - Key points

- Noise (signal to noise ratio) -> relation to experimental parameters
- SNR in magnitude and phase domain
- Contrast, its optimization
- Partial voluming
- Affect of common exogenous contrast agents

Noise

Any physical measurement using an instrument/sensor has noise and it is our ability to detect the signal above the noise that characterizes a successful measurement



A measure of a successful experiment is the ratio

$$\frac{Signal}{Noise\ standard\ deviation}$$

$$\text{SNR} = \frac{S}{\sigma_0}$$

Intro

- Noise is practically inevitable
  - Random / systematic
  - Physiological
- Signal-to-noise ratio (SNR) determines the effectiveness of the imaging
- SNR affects CNR, which determines the usefulness of the image
- Good CNR requires good SNR, but good SNR not necessary means good CNR

Signal and noise

- **Consider equilibrium magnetization and receive coil field**

$$s \equiv \hat{\rho}_m \propto \omega_0 M_0 \mathcal{B}_\perp V_{voxel}$$

$$\propto \gamma B_0 \rho_0 \frac{\gamma^2 h^2}{4kT} B_0 \mathcal{B}_\perp V_{voxel}$$

$$\propto \frac{B_0^2 V_{voxel}}{T} \rho_0 \mathcal{B}_\perp$$

- **Many factors being ignored or constant here**
  - **T1, T2, T2***
  - **RF field: flip angle, B1 field homogeneity**
  - **Sequence type, TE/TR/TI/…**

Signal and noise

- Revisiting the measured signal

1D:

$$\hat{\rho}_m(q\Delta x) = \frac{1}{\mathrm{N}}\sum_{p=-\frac{N}{2}}^{\frac{N}{2}-1} s(p\Delta k)e^{i2\pi pq/N}$$

$$= \Delta x \sum_{q=-\frac{N}{2}}^{\frac{N}{2}-1} \hat{\rho}(q\Delta x)\delta(x - q\Delta x)$$

3D:

Voxel Signal

$$\hat{\rho}_m(p\Delta x, q\Delta y, r\Delta z) = \frac{1}{N_x N_y N_z}\sum_{p',q',r'} s(p'\Delta k_x, q'\Delta k_y, r'\Delta k_z)e^{i2\pi\left(\frac{pp'}{N_x}+\frac{qq'}{N_y}+\frac{rr'}{N_z}\right)}$$

Voxel Volume

$$= \Delta x\Delta y\Delta z \sum_{p,q,r} \hat{\rho}(p\Delta x, q\Delta y, r\Delta z)\delta(x - p\Delta x)\delta(y - q\Delta y)\delta(z - r\Delta z)$$

Signal and noise

- **Noise in MRI**
  - **Thermal noise (body/object, coil, electronics)**
  - **Systematic noise (scanner, AC power, etc)**
  - **Physiological noise (heartbeat, respiration, etc)**

- **Thermal noise estimation**

$$\sigma_{thermal}^2 = \sigma_{body}^2 + \sigma_{coil}^2 + \sigma_{electronics}^2$$
$$\propto 4kT \cdot R \cdot BW_{coil}$$

$$R_{eff} = R_{body} + R_{coil} + R_{electronics}$$

$BW_{coil} \equiv BW_{readout}$: **bandwidth of reception**

Note that this is noise in k-space domain – What is relevant for us is the noise in image domain

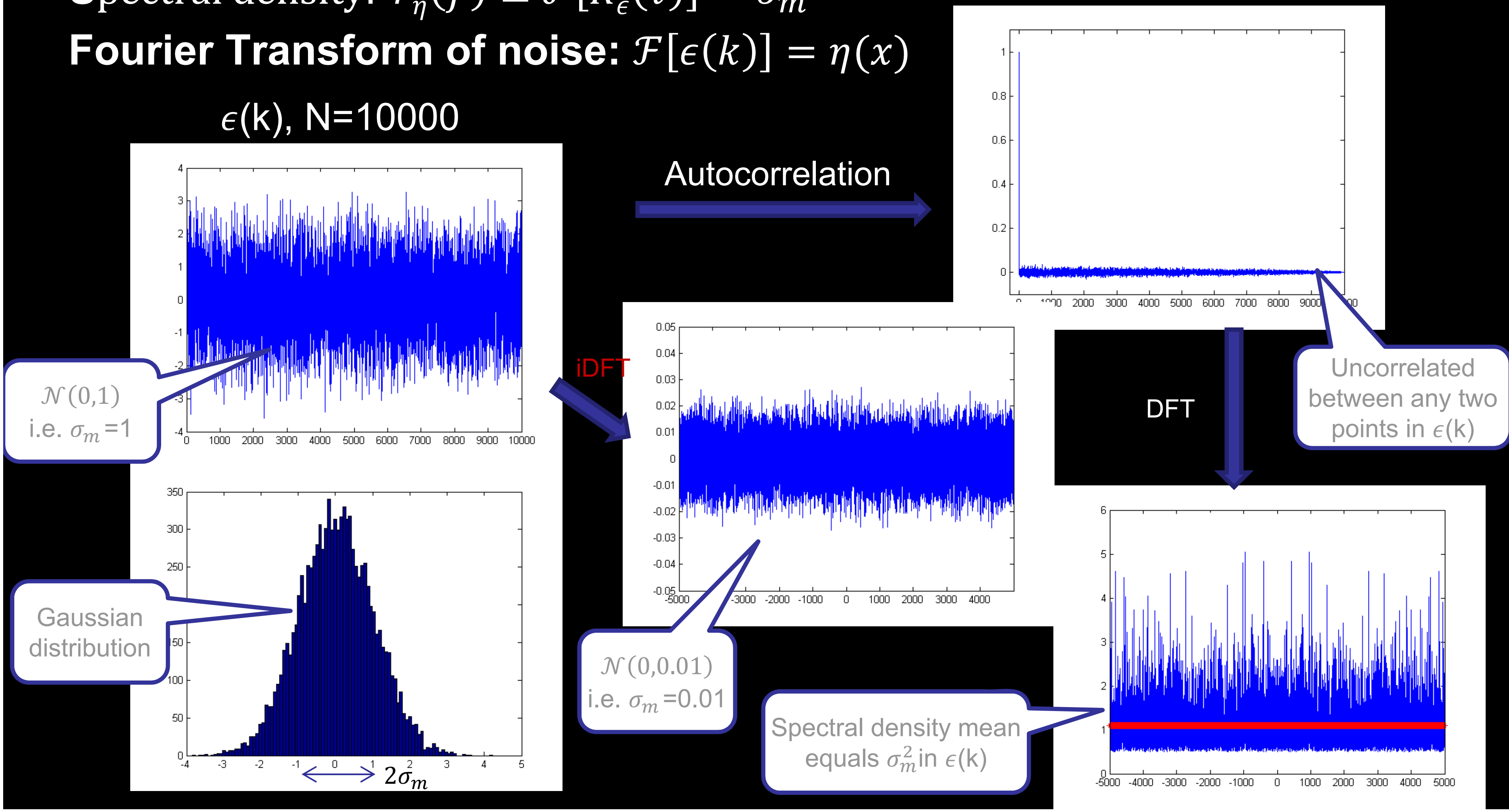
$$s_m(k) = s(k) + \epsilon(k)$$

- **For thermal (or white/Gaussian) noise**

Autocorrelation: $R_\epsilon(\tau) \equiv \overline{\epsilon(k_p)\epsilon^*(k_q)}|_{\tau\equiv(k_p-k_q)} = \sigma_m^2\delta(\tau)$

Spectral density: $r_\eta(f) \equiv \mathcal{F}[R_\epsilon(\tau)] = \sigma_m^2$

Fourier Transform of noise: $\mathcal{F}[\epsilon(k)] = \eta(x)$



Noise vs. Imaging parameters

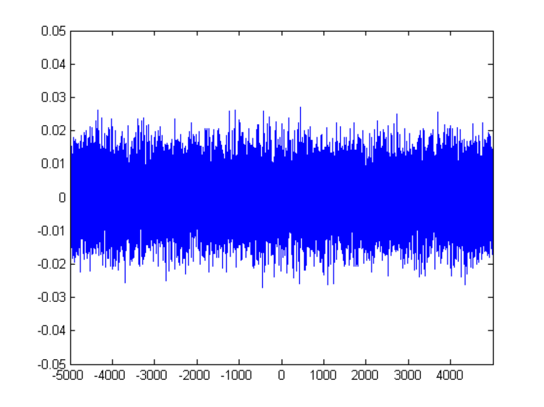
- **Noise in MR image**

$$\eta(p\Delta x) = iDFT[\epsilon(k)] = \frac{1}{N}\sum\nolimits_{p'} \epsilon(p'\Delta k) e^{i2\pi p'\Delta k p \Delta x}$$

- **Properties of noise in image**

$$mean\big(\eta(p\Delta x)\big) = iDFT\big[\overline{\epsilon(k)}\big] = 0$$

$$var\big(\eta(p\Delta x)\big) \equiv DFT[R_\epsilon(\tau)] = \frac{\sigma_m^2}{N}$$



- **Implications**
  - White noise in k-space results in white noise in image
  - Noise variance (std) in image is N ($\sqrt{N}$) times smaller than in k-space

Improving SNR

- **Averaging over multiple repeated acquisitions**

$$s_{m,ave}(k) = \frac{1}{N_{acq}} \sum_i s_{m,i}(k) = s_m(k)$$

$$\sigma_{m,ave} = \sqrt{\frac{\sum_i \sigma_{m,i}^2}{N_{acq}}} = \frac{\sigma_m}{\sqrt{N_{acq}}}$$

Thus

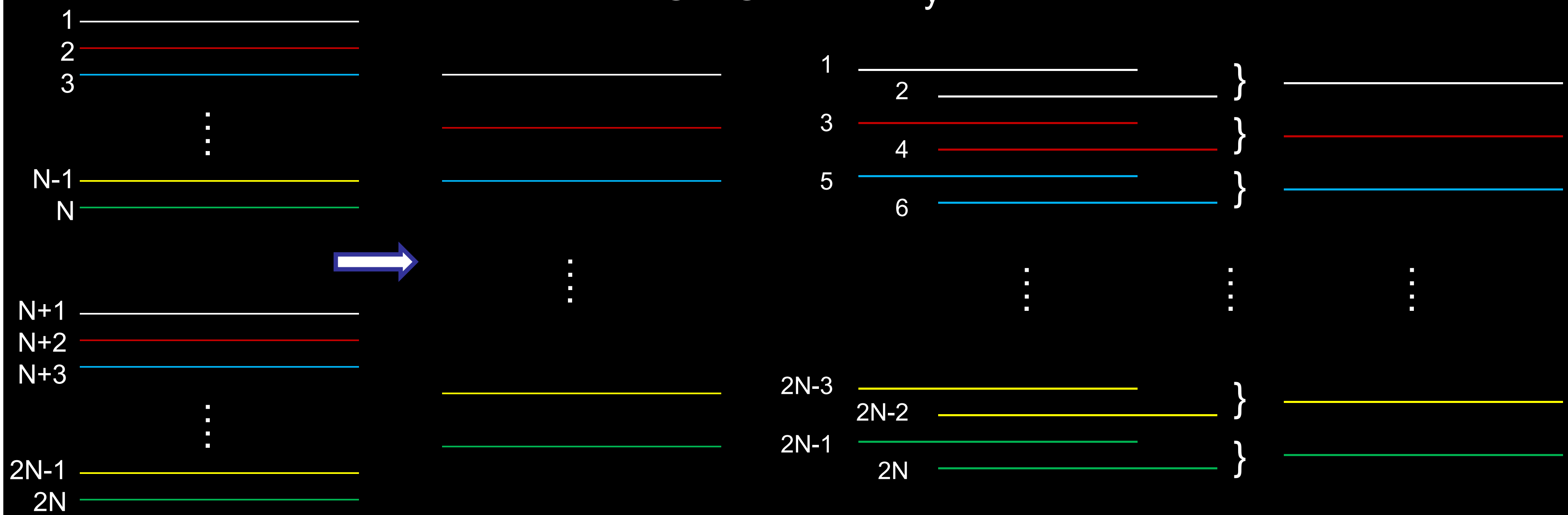
$$SNR_{m,ave} = \sqrt{N_{acq}} SNR_m$$

- **Properties**
  - **True for both k-space and images**
  - **Valid only when noise are uncorrelated between acquisitions, systematic noise such as artifacts will not be reduced**

Improving SNR: Multiple averages

Improving SNR

- Averaging can be done over images or k-space
- Two modes for averaging over k_y



Long term ave

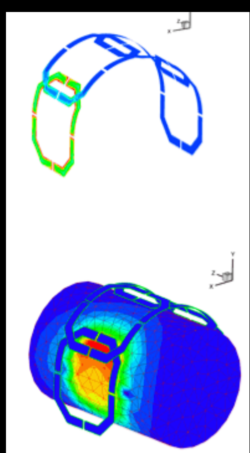
Short term ave

$$\sigma_{0,avg} = \frac{\sigma_0}{\sqrt{N_{acq}}}$$

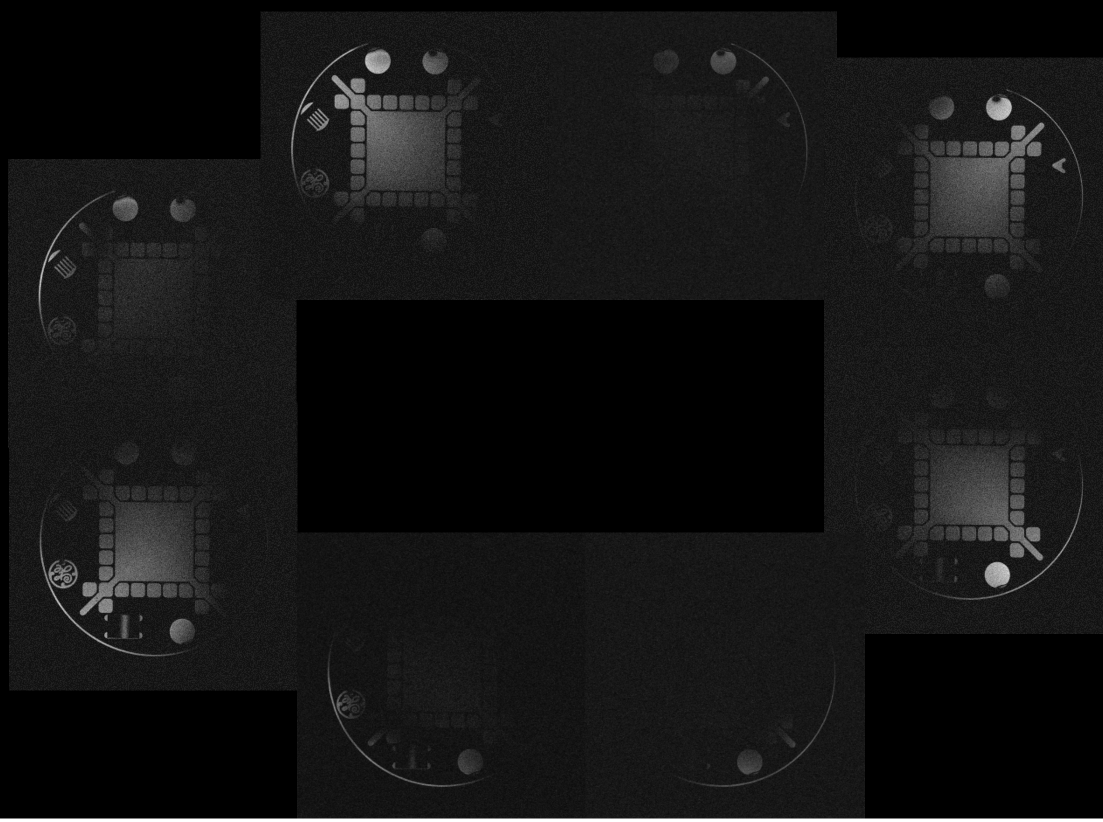
$$\boxed{SNR_{ave} = \sqrt{N_{acq}} \cdot SNR}$$

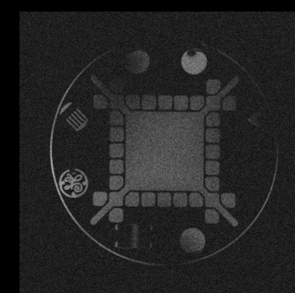
Improving SNR

- **Use multiple smaller surface coils (phased array coils)**
  - **Smaller coil -> less coupling -> less noise**
  - **Rapidly weakening receive fields $\mathcal{B}$ with distance**
  - **Sum-of-square offers optimal results over linear combination**

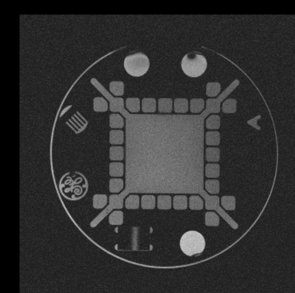


http://www.insightmri.com/





Linear



SOS

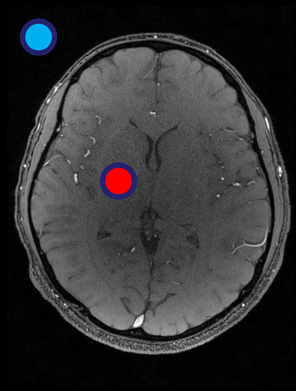
Measuring SNR

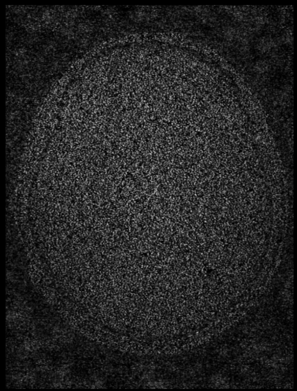
- Mean and std of ROI in homogeneous regions
- Use std in background ROI to estimate σ_0 ($\sigma_{bg} = 0.655\ \sigma_0$, Rayleigh distribution)

$\sigma_{oj} > \sigma_{bg}$

Gaussian vs. Rayleigh;

- Scan twice, add and subtract the two images to get mean and std ($\sigma_{sub} = \sqrt{2}\sigma_0$)
- Collect multiple volumes of images for voxel wise SNR (tSNR)



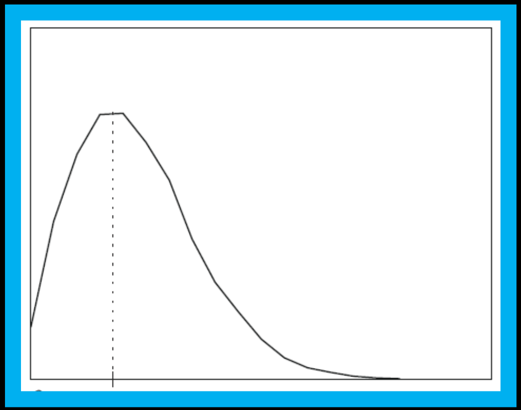


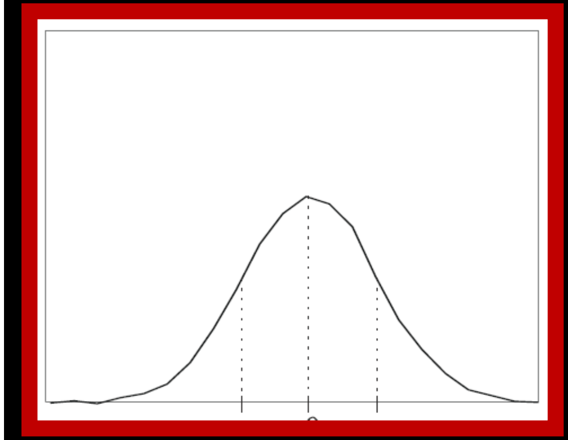
subtracted

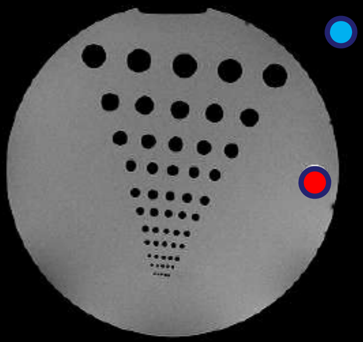
Measuring SNR

($\sigma_{bg} = 0.655\ \sigma_0$, Rayleigh distribution)

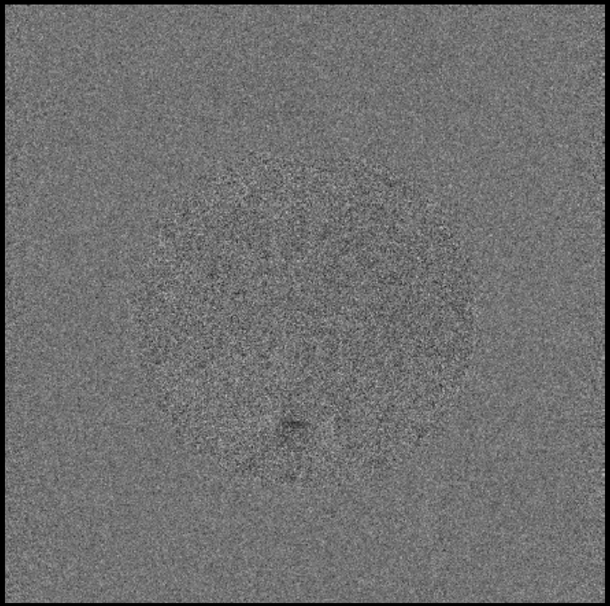
$\sigma_{oj} > \sigma_{bg}$







Single Acq



subtracted

SNR dependence on imaging parameters

Voxel signal ∝ Δx (Voxel volume)

Noise standard deviation $\sigma \propto \frac{1}{\sqrt{N_x}}$

Noise standard deviation $\sigma \propto \frac{1}{\sqrt{N_{acq}}}$

Noise standard deviation $\sigma \propto \sqrt{BW_{readout}}$

$$SNR/voxel \equiv \frac{Signal_{voxel}}{\sigma} \propto \frac{\Delta x \Delta y \Delta z \sqrt{N_{acq}}}{\sqrt{\frac{BW_{read}}{N_x N_y N_z}}}$$

SNR dependence on imaging parameters

$$SNR/voxel \equiv \frac{Signal_{voxel}}{\sigma} \propto \frac{\Delta x \Delta y \Delta z \sqrt{N_{acq}}}{\sqrt{\frac{BW_{read}}{N_x N_y N_z}}}$$

$\frac{1}{\Delta t} = \bar{\gamma} \cdot G_x \cdot N_x \cdot \Delta x$ Total read bandwidth

$$\frac{SNR}{voxel} \propto \Delta x \Delta y \Delta z \sqrt{N_{acq} N_x N_y N_z \Delta t}$$

$$T_s = N_x \Delta t$$

$$\frac{SNR}{voxel} \propto \Delta x \Delta y \Delta z \sqrt{N_{acq} N_y N_z T_s}$$

$$\text{SNR/voxel} \propto \frac{\Delta x \Delta y \Delta z \sqrt{N_{acq}}}{\sqrt{\frac{BW_{read}}{N_x N_y N_z}}}$$

$$\text{SNR/voxel} \propto \Delta x \Delta y \Delta z \sqrt{N_{acq} N_x N_y N_z \Delta t}$$

$$\text{SNR/voxel} \propto \Delta x \Delta y \Delta z \sqrt{N_{acq} N_y N_z T_s}$$

(a) $L_x = N_x \Delta x$

(b) $L_y = N_y \Delta y$

(c) $L_z = N_z \Delta z$

(d) $T_s = N_x \Delta t$

(e) $BW_{read} = \frac{1}{\Delta t} = \gamma G_x L_x$

(f) $BW/\text{voxel} = BW_{read}/N_x$

Case	Δx	N_x	L_x	G_x	Δt	T_s	SNR
Reference case							
1	Δx_0	N_0	L_0	G_0	Δt_0	$T_{s,0}$	1
6	$\Delta x_0/2$	N_0	$L_0/2$	G_0			
7	$\Delta x_0/2$	N_0	$L_0/2$	$2G_0$			

$$\text{SNR/voxel} \propto \frac{\Delta x \Delta y \Delta z \sqrt{N_{acq}}}{\sqrt{\frac{BW_{read}}{N_x N_y N_z}}}$$

$$\text{SNR/voxel} \propto \Delta x \Delta y \Delta z \sqrt{N_{acq} N_x N_y N_z \Delta t}$$

$$\text{SNR/voxel} \propto \Delta x \Delta y \Delta z \sqrt{N_{acq} N_y N_z T_s}$$

(a)	$L_x = N_x \Delta x$	(b)	$L_y = N_y \Delta y$
(c)	$L_z = N_z \Delta z$	(d)	$T_s = N_x \Delta t$
(e)	$BW_{read} = \frac{1}{\Delta t} = \gamma G_x L_x$	(f)	$BW/\text{voxel} = BW_{read}/N_x$

Case	Δx	N_x	L_x	G_x	Δt	T_s	SNR
Reference case							
1	Δx_0	N_0	L_0	G_0	Δt_0	$T_{s,0}$	1
6	$\Delta x_0/2$	N_0	$L_0/2$	G_0	$2\Delta t_0$	$2T_{s,0}$	$1/\sqrt{2}$
7	$\Delta x_0/2$	N_0	$L_0/2$	$2G_0$	Δt_0	$T_{s,0}$	$1/2$

SNR dependence on imaging parameters

$$SNR/voxel \equiv \Delta y \Delta z \sqrt{N_y N_z}$$

$$\frac{SNR}{voxel} \propto \Delta x \Delta y \Delta z \sqrt{N_{acq} N_y N_z T_s}$$

2D vs 3D

$$\Delta z \equiv TH$$

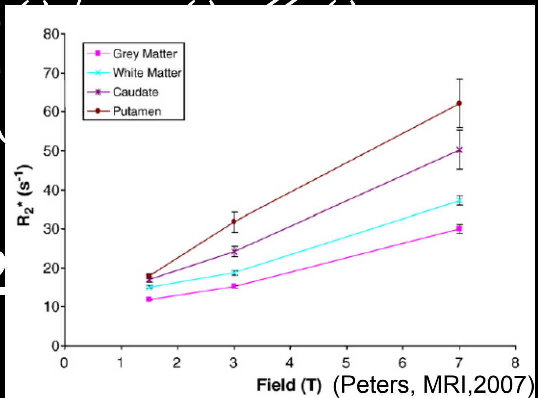
$$(\mathrm{SNR}/\mathrm{voxel})|_{2D} \propto \Delta x \Delta y TH \sqrt{N_y T_s}$$

SNR vs. B_0

- B_0 affects MRI signal and noise in a number of ways, e.g. M_0, T1, T2, field inhomogeneity, and thus T2*
  - $S \propto \frac{B_0^2 V_{voxel}}{T} \propto \omega_0^2$
  - $R_{eff}(\omega_0) \approx \begin{cases} R_{coil}(\omega_0) + R_{electronices}(\omega_0) \\ R_{sample}(\omega_0) \end{cases} \propto \begin{cases} \sqrt{\omega_0}, & \omega_0 \ll \omega_{0,mid} \\ \omega_0^2, & \omega_0 \gg \omega_{0,mid} \end{cases}$

$$SNR(\omega_0) \propto \begin{cases} \omega_0^{7/4} \\ \omega_0 \end{cases} \longleftarrow \sigma_{thermal} \propto \begin{cases} \omega_0^{1/4} \\ \end{cases}$$

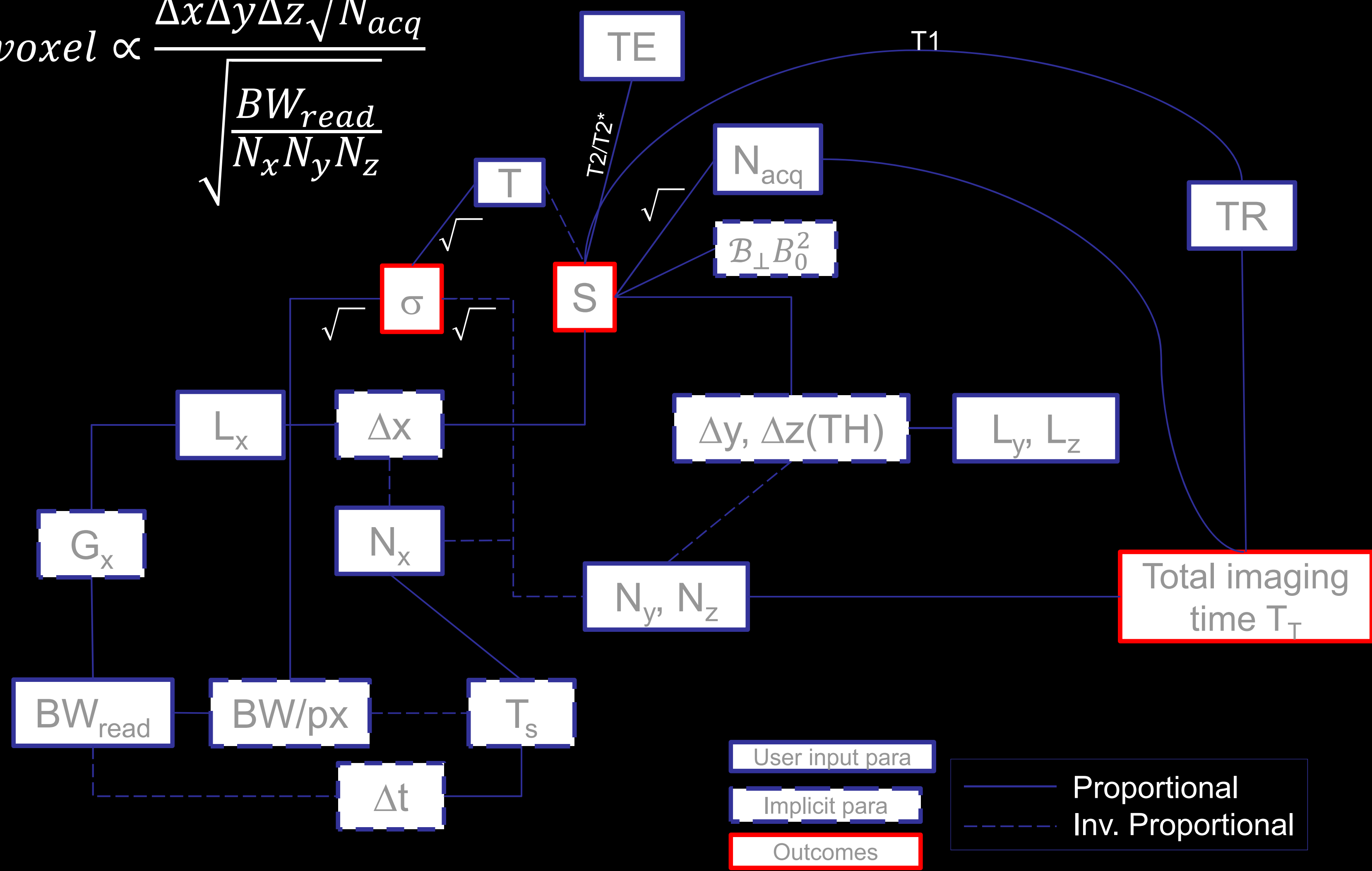
- **Higher B_0 -> longer T1, shorter T2/T2**



(Peters, MRI,2007)

SNR dependence on imaging parameters

$$SNR/voxel \propto \frac{\Delta x \Delta y \Delta z \sqrt{N_{acq}}}{\sqrt{\frac{BW_{read}}{N_x N_y N_z}}}$$



Imaging efficiency

- **Total imaging time**

$$T_T = N_{acq} N_y N_z TR$$

- **Imaging efficiency (with otherwise fixed parameters)**

$$\Upsilon \equiv \frac{SNR/voxel}{\sqrt{T_T}} \propto \Delta x \Delta y \Delta z \sqrt{T_s}$$

- **Implications**
  - Large voxels yield better SNR efficiency
  - So does longer readout (or lower RO bandwidth). But sometimes T2* decay may negates the effects

Contrast, CNR and visibility

- High SNR is only half the story

- The other half is to be able to distinguish different object/structure/properties, i.e. have contrast with the presence of noise

Contrast, CNR and visibility

- **Contrast: simply the signal difference**

$$C_{AB} \equiv S_A - S_B$$

- **CNR: contrast-to-noise ratio**

$$CNR_{AB} \equiv SNR_A - SNR_B$$

when $\sigma_A = \sigma_B = \sigma_0$,

$$CNR_{AB} = \frac{C_{AB}}{\sigma_0}$$

Contrast, CNR and visibility

- **Visibility**

$$\mathcal{V}_{AB} \equiv \frac{C_{AB}}{\sigma_{eff}} = \frac{C_{AB}}{\sigma_0}\underbrace{\sqrt{N_{acq}}}_{\text{Temporal}} = \frac{C_{AB}}{\sigma_0}\underbrace{\sqrt{n_{voxel}}}_{\text{Spatial}}$$

- **The Rose Criterion**

$$CNR_{AB} > 3\sim5$$

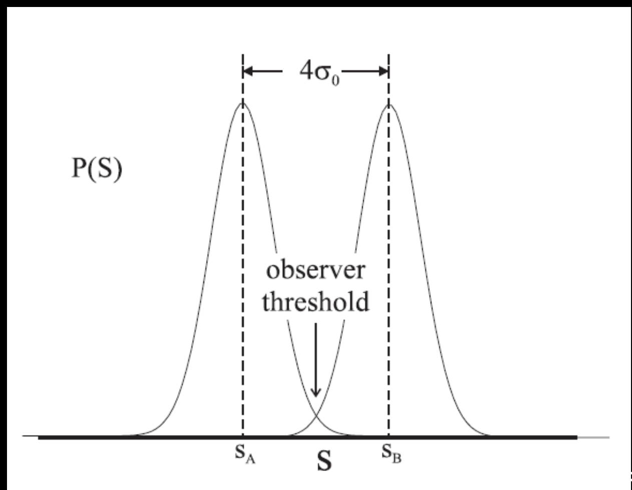
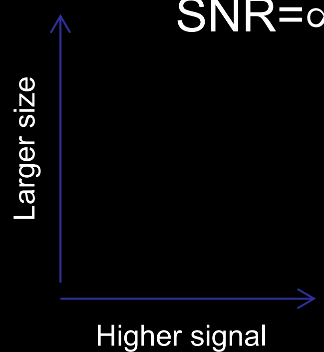


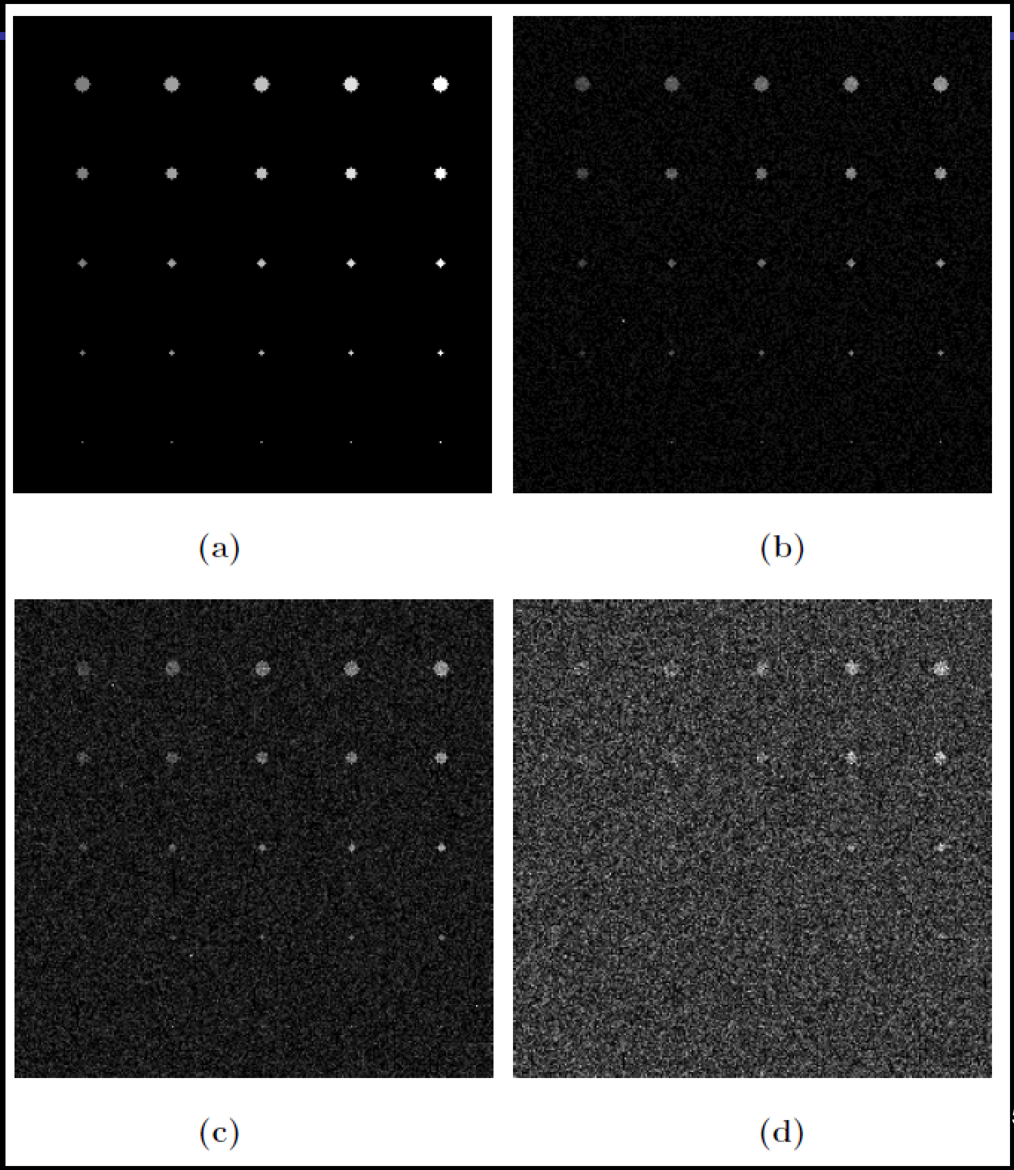
Fig. 5.4

Contrast, CNR and visibility



SNR=∞

SNR=4



SNR=2

SNR=1

Homework

- Prob 15.1, 15.3, 15.4, 15.8, 15.10, 15.12, 15.13

Next Session

Chapter 15.3-15.6

Contrast mechanisms in MRI

- **MRI is a multiple contrast imaging modality**

Intrinsic:
- spin density
- T1/T2/T2*/T1ρ
- Diffusion
- Chemical shift
- Susceptibility
- Temperature
- …

Physiological:
- Blood flow
- Perfusion
- Blood oxygenation
- Tissue elasticity
- …

Sequence specific:
- Steady state (chap.18)
- Selective excitation /suppression
- …

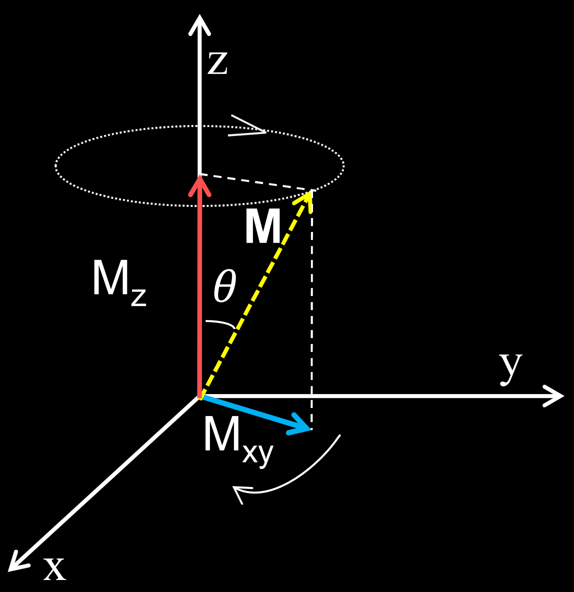
- **Or multiple methods for the same contrast**
  - **Example:**
    T1W : short TE/TR GE, or IR
    Signal nulling: IR, selective excitation/saturation, dephasing

Basic Contrast Mechanisms: Relaxation effects

$$\frac{d\vec{M}}{dt} = \gamma \cdot \vec{M} \times B + \frac{M_0 - M_z}{T_1} - \frac{M_{xy}}{T_2}$$

M_0 is the equilibrium magnetization

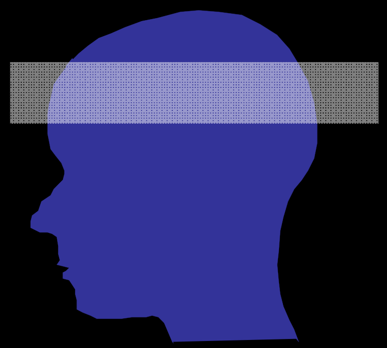
M_{xy} is the transverse magnetization

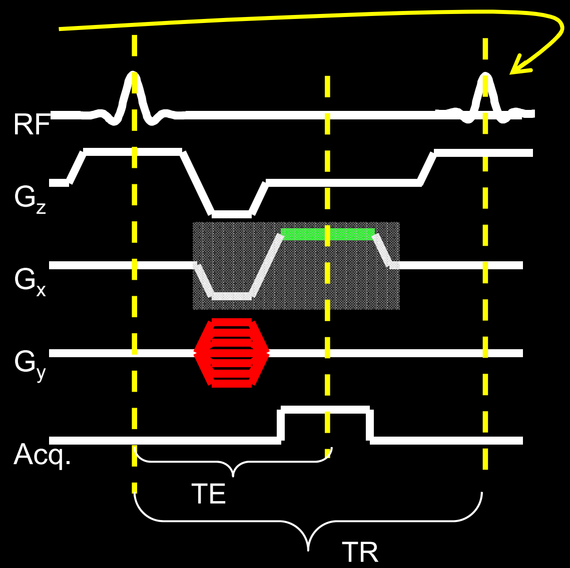


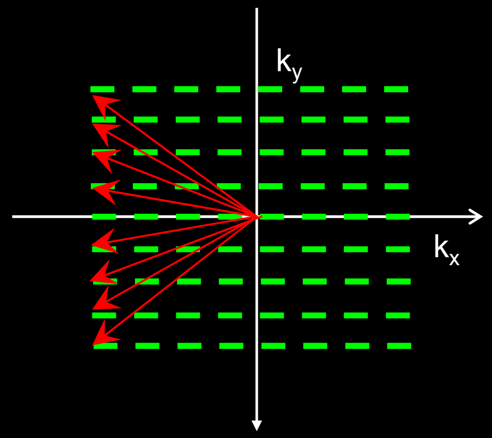
Contrast

Tissue	T_1 (ms)	T_2 (ms)
gray matter (GM)	950	100
white matter (WM)	600	80
muscle	900	50
cerebrospinal fluid (CSF)	4500	2200
fat	250	60
blood[3]	1200	100-200[4]

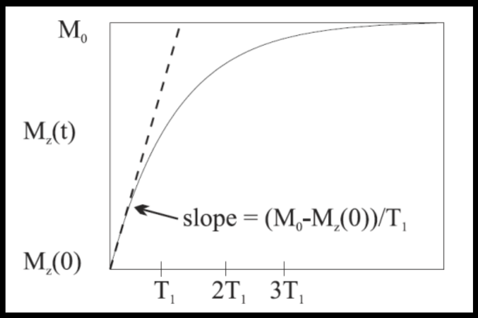
Contrast





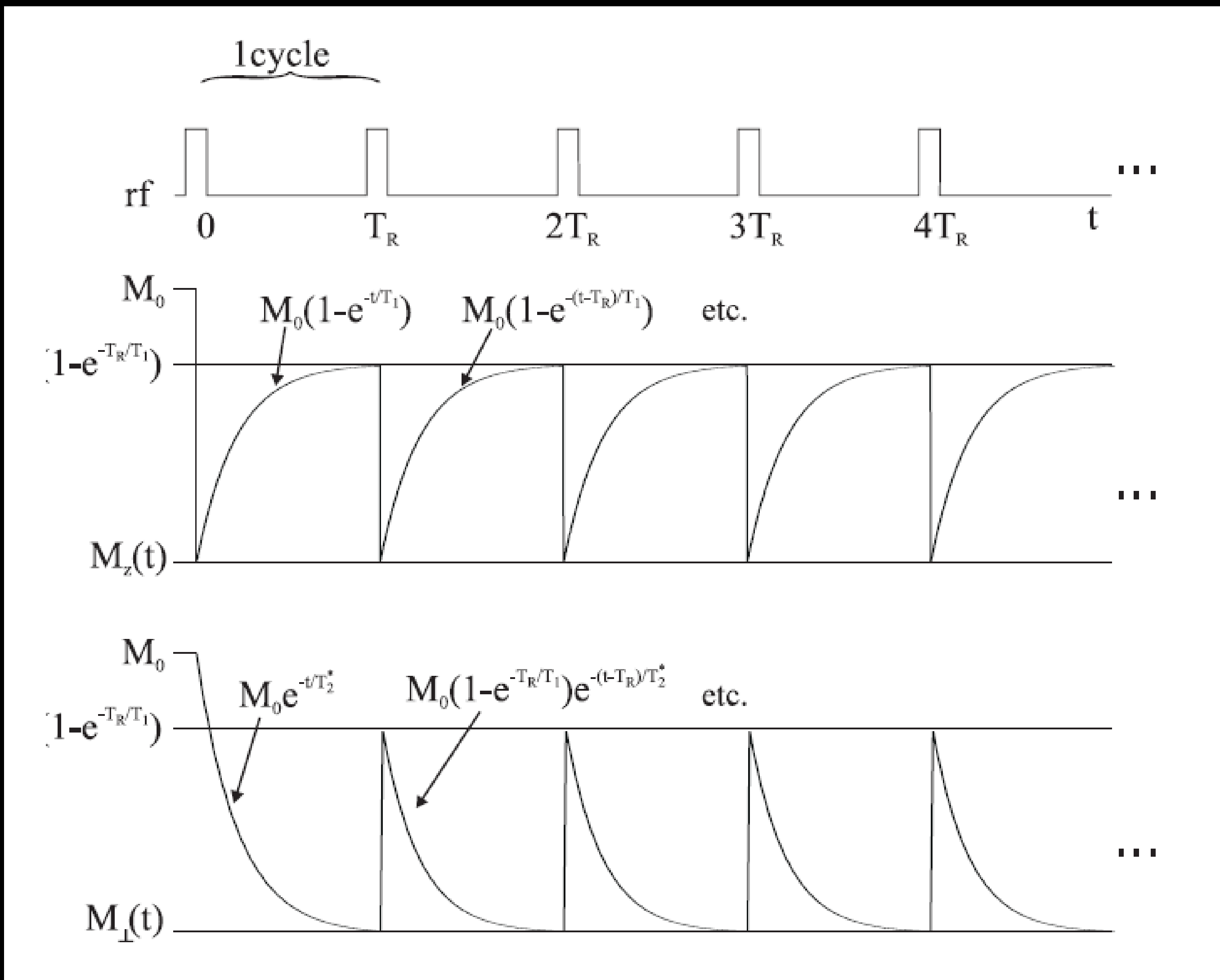


Once you tip the magnetization of a tissue, it takes a finite amount of time for it to recover

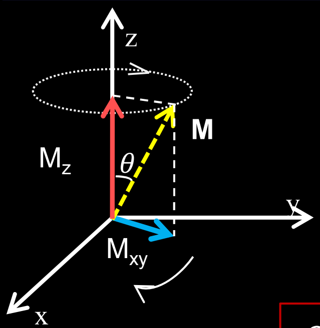


With repeated rf pulse excitations during image acquisition, the time intervals of repetition and acquisition (TR,TE), along with T1 and T2 values of different tissue determine the final contrast

Magnetization equations



Magnetization equations



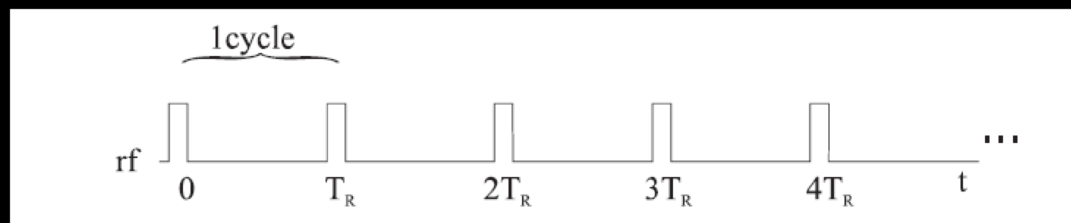
$$M_z(t) = M_z(0)\cdot e^{-\frac{t}{T_1}} + M_0\cdot\left(1 - e^{-\frac{t}{T_1}}\right)$$

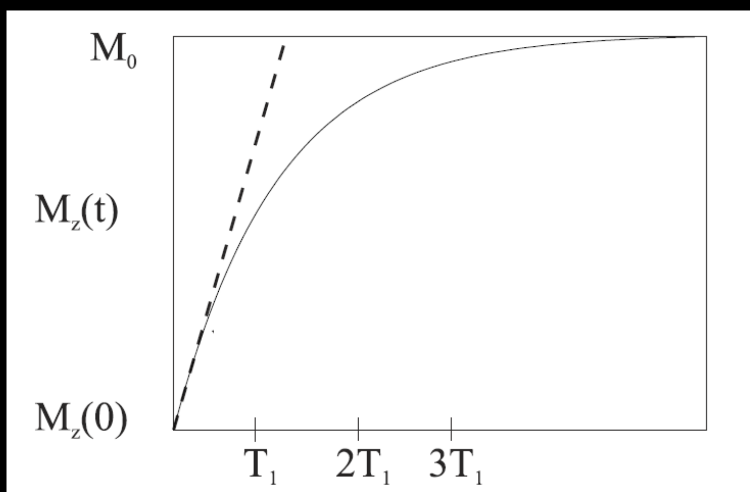
$$M_{xy}(t) = M_{xy}(0)\cdot e^{-\frac{t}{T_2}}$$

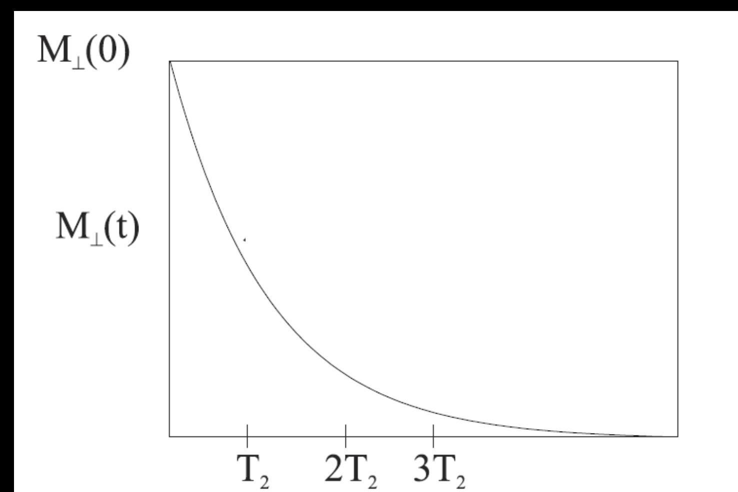
$$M_z(0) = M_0\cdot\cos(\theta)$$

$$M_{xy}(0) = M_0\cdot\sin(\theta)$$

$$\theta = 90^o$$







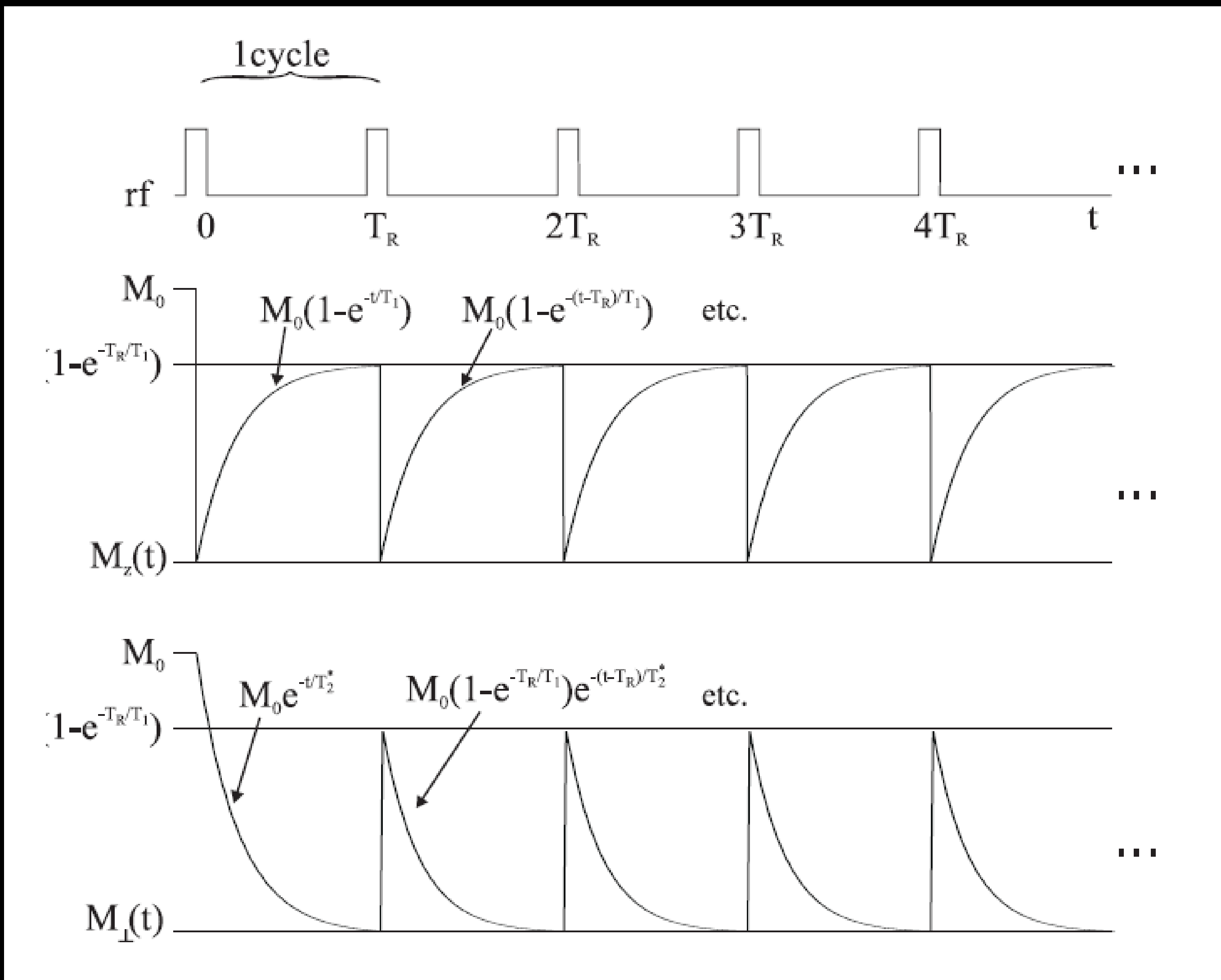
$$M_z(TR^-) = M_0\cdot\left(1 - e^{-\frac{TR}{T_1}}\right)$$

$$M_{xy}(TE) = M_0\cdot\left(1 - e^{-\frac{TR}{T_1}}\right)\cdot e^{-\frac{TE}{T_2}}$$

$$M_{xy}(t) = M_{xy}(0)\cdot e^{-\frac{TR}{T_2}}$$

$$TR \gg T_2$$

Magnetization equations

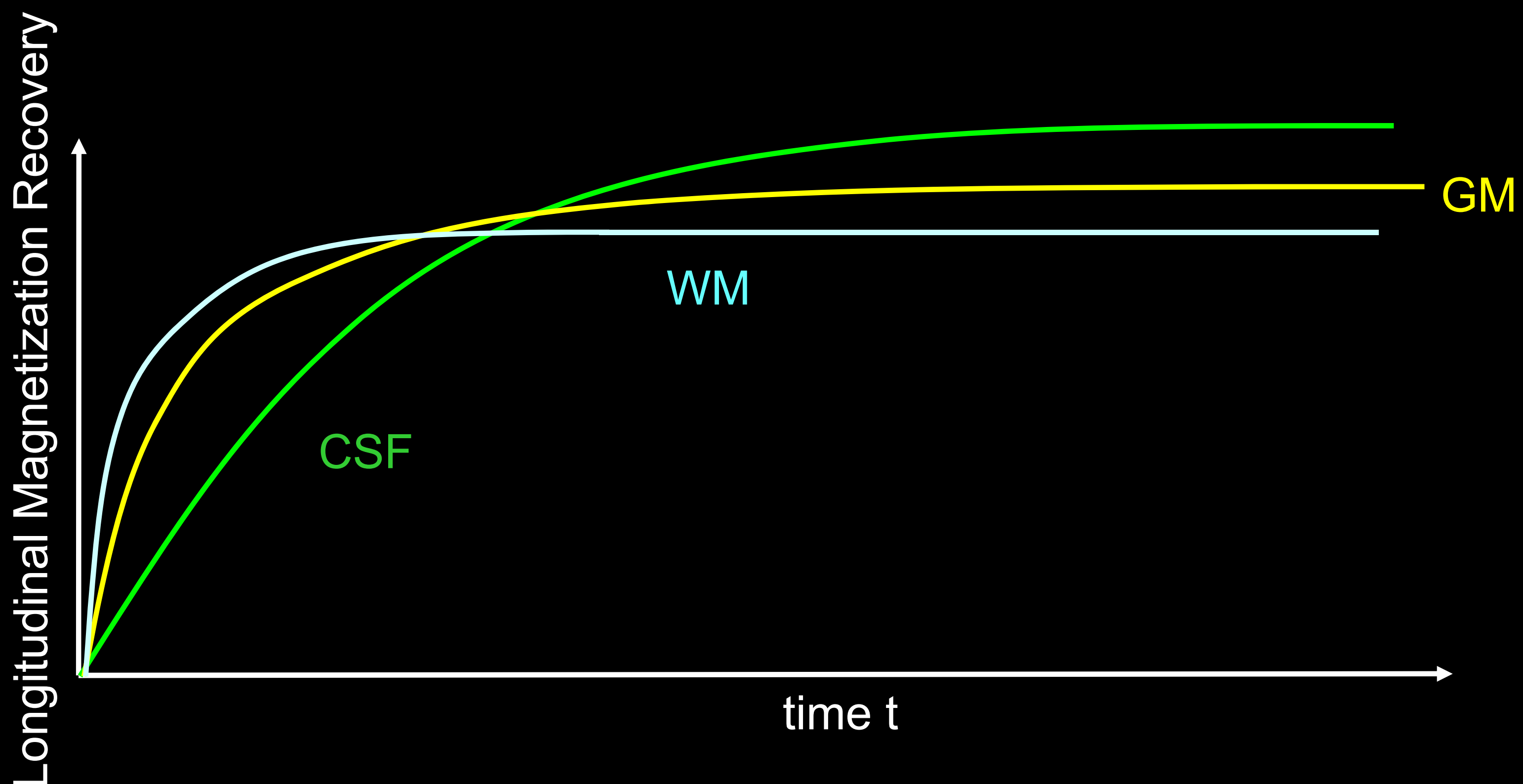


Take-aways from the math

Steady state reached by the second rf pulse

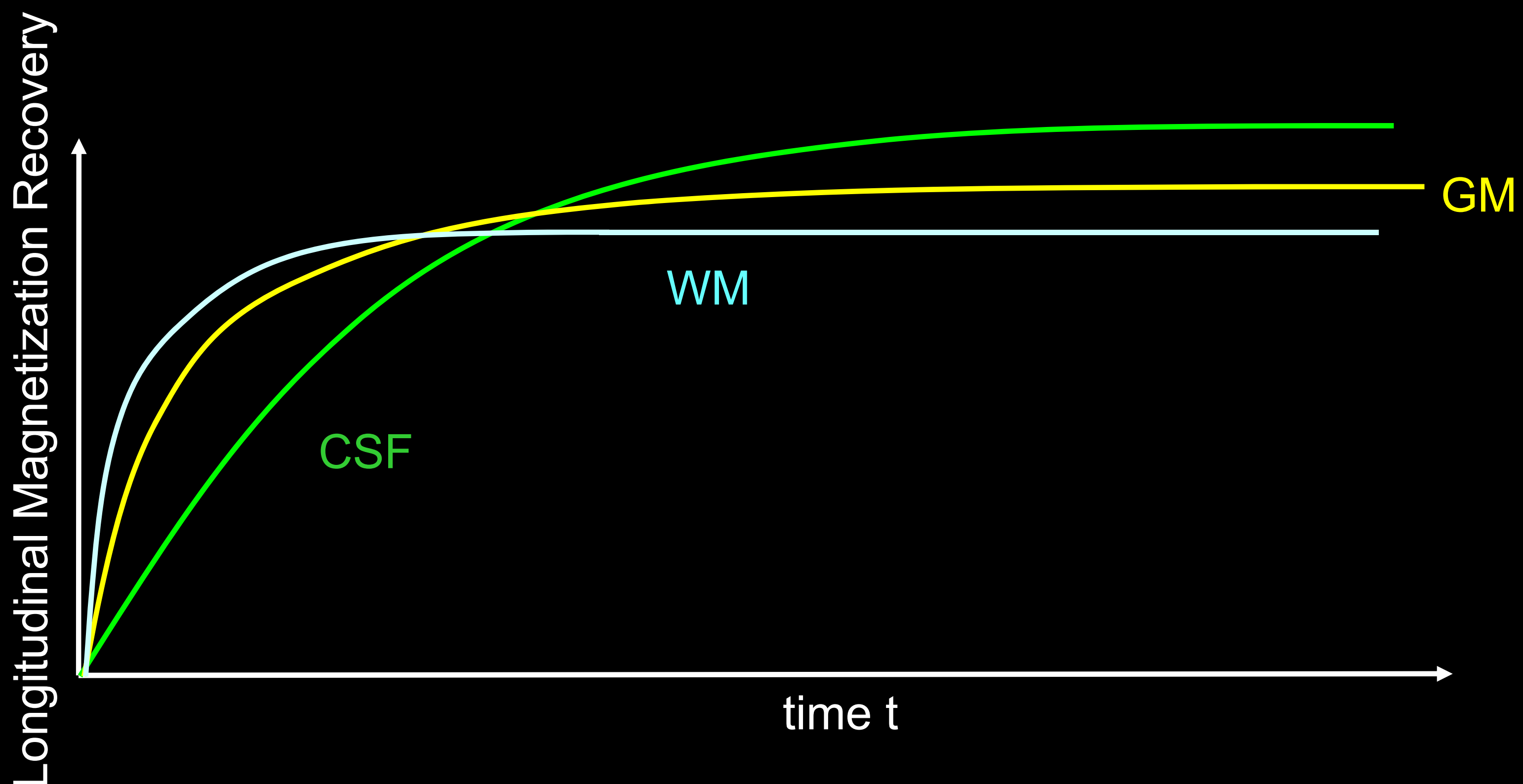
Steady state meaning… $M_z(TR)$ and $M_{xy}(TE)$ are constant

Contrast

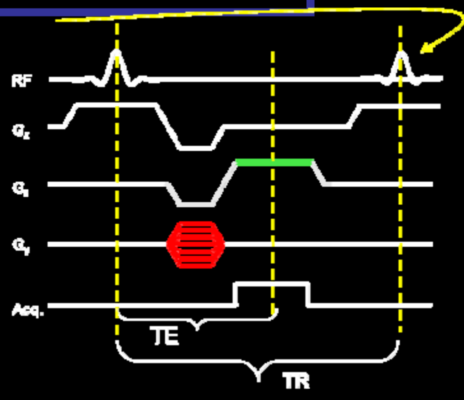


Contrast

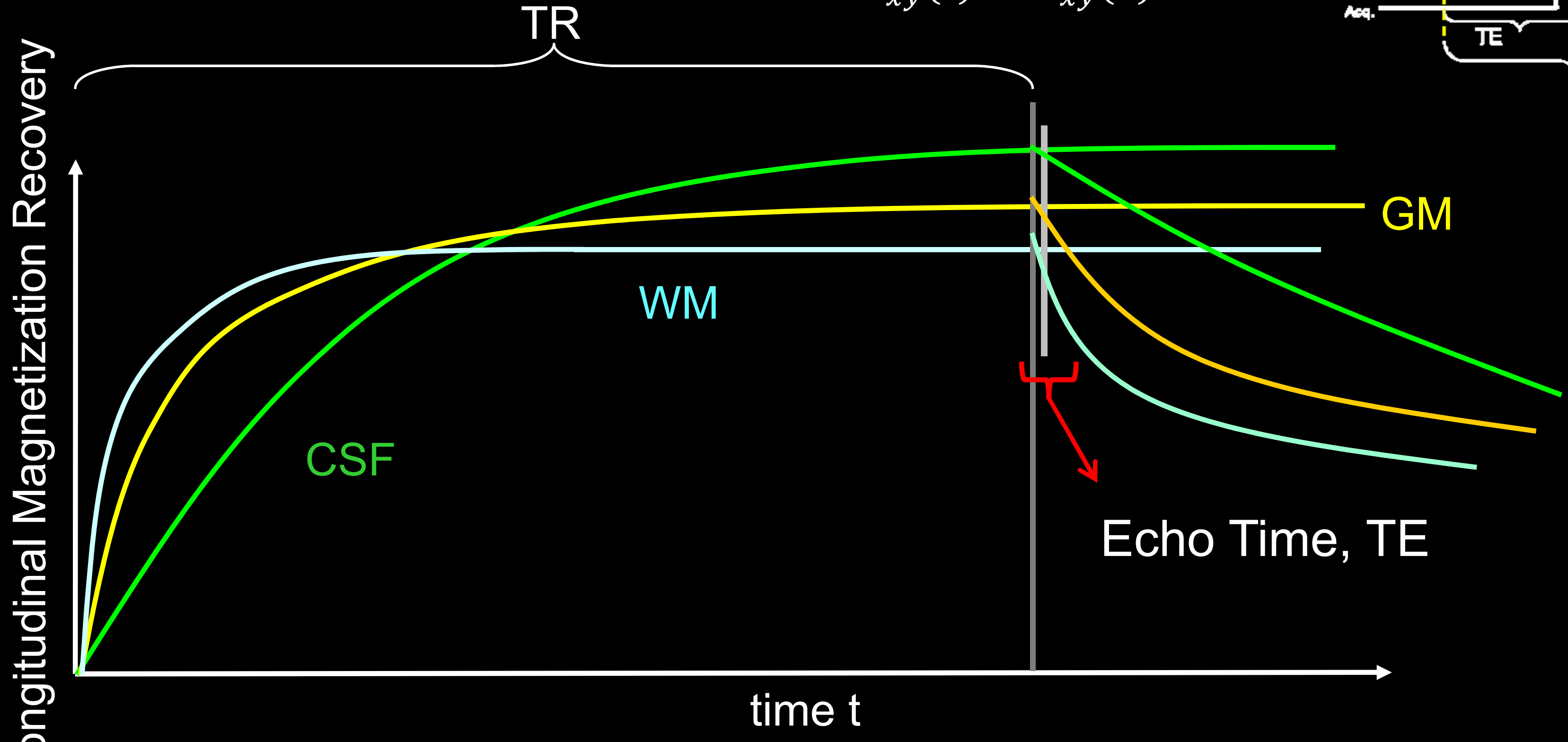
Different tissues have different amounts of water and hence different densities of ^{1}H protons within them.



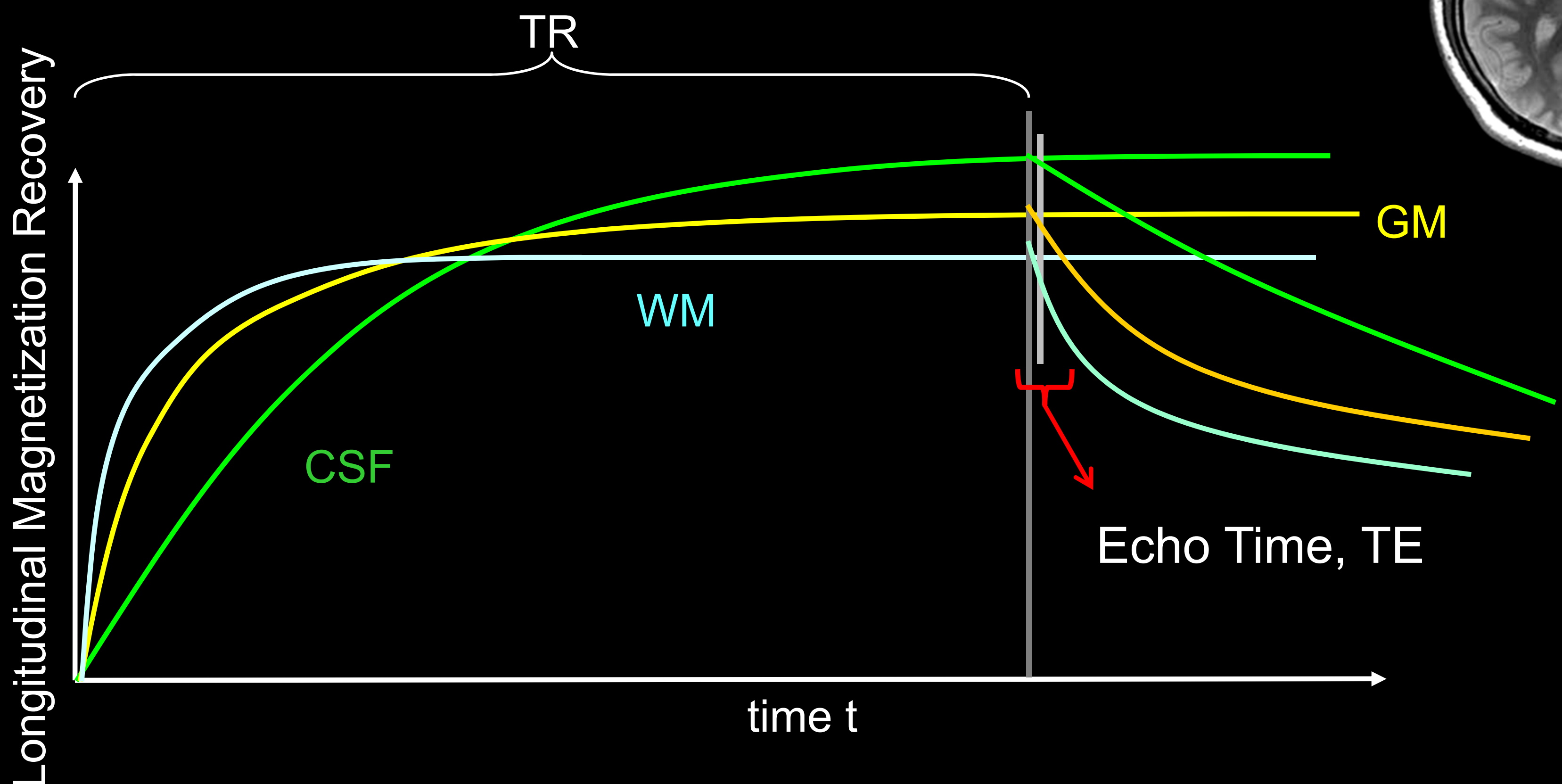
Contrast

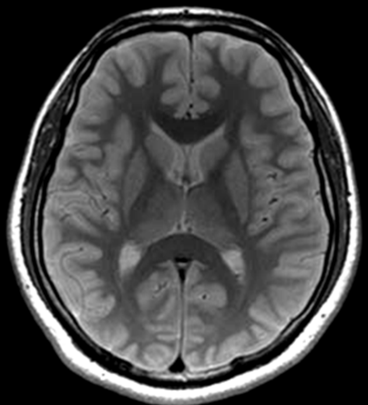


$$M_{xy}(t) = M_{xy}(0) \cdot e^{-\frac{TE}{T_2}}$$



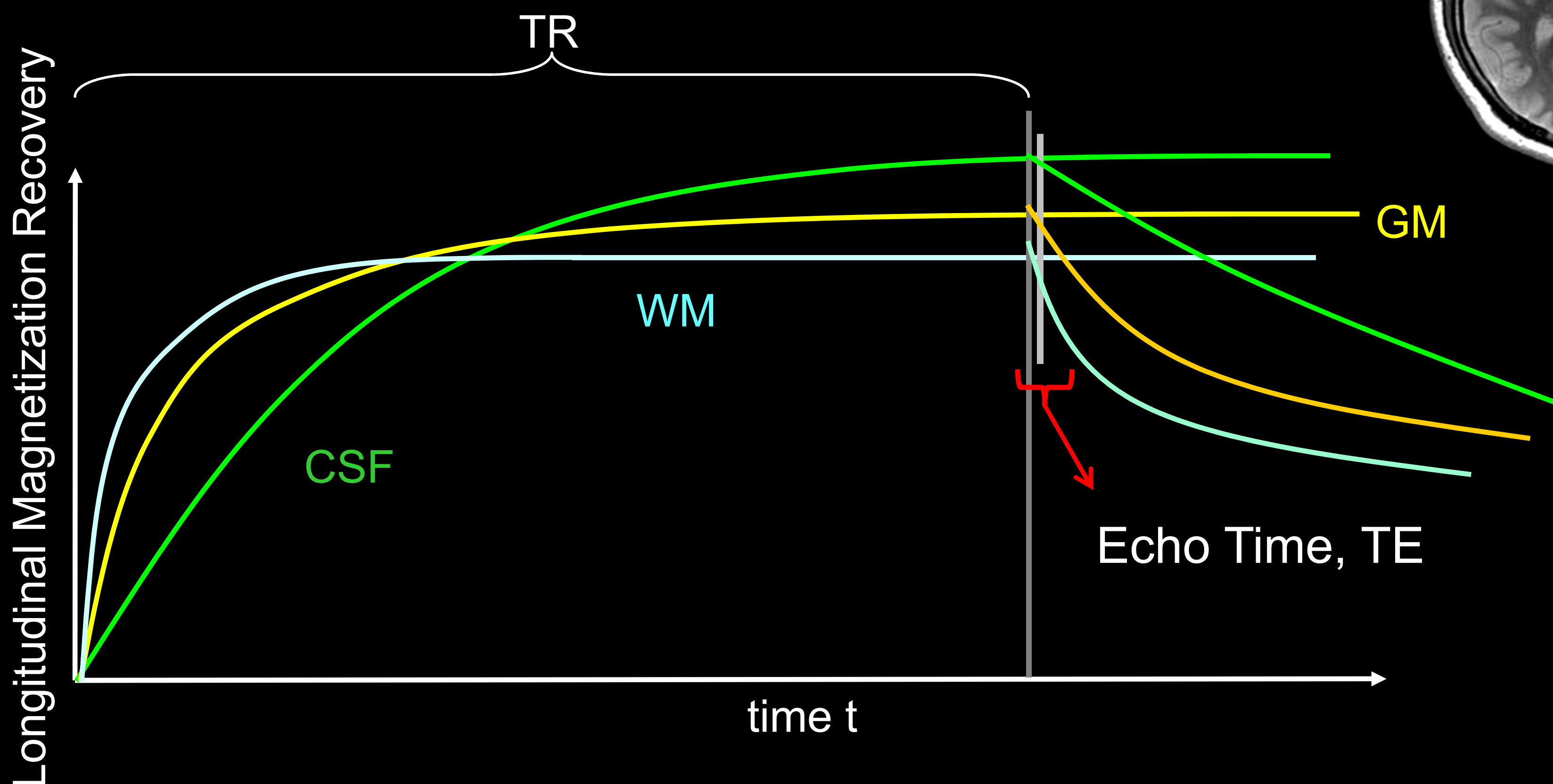
Contrast…ρ_0 weighting

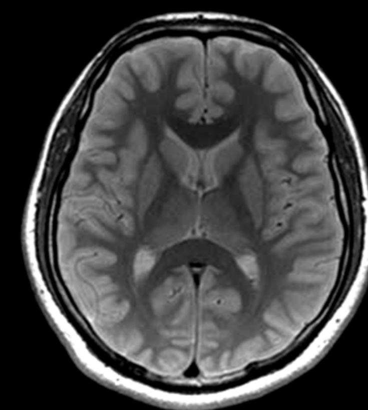




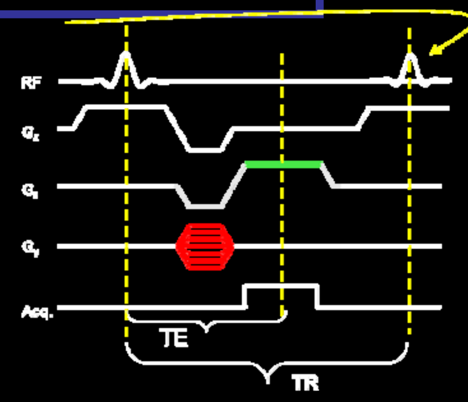
Contrast…ρ_0 weighting

Long repetition time -TR
Short echo time - TE

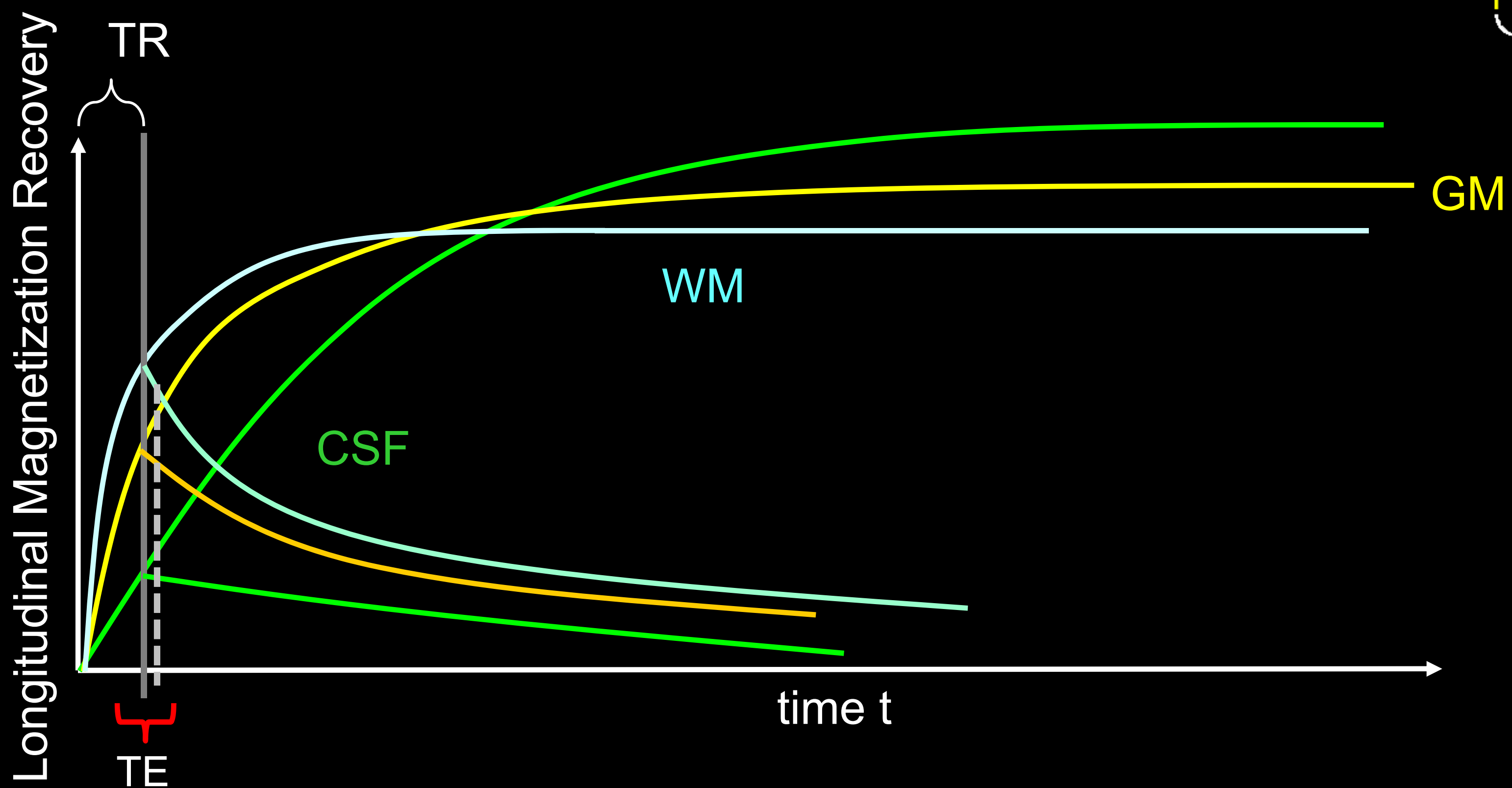




Contrast



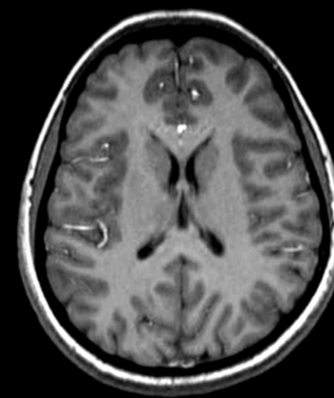
RF
Acq.
TE
TR

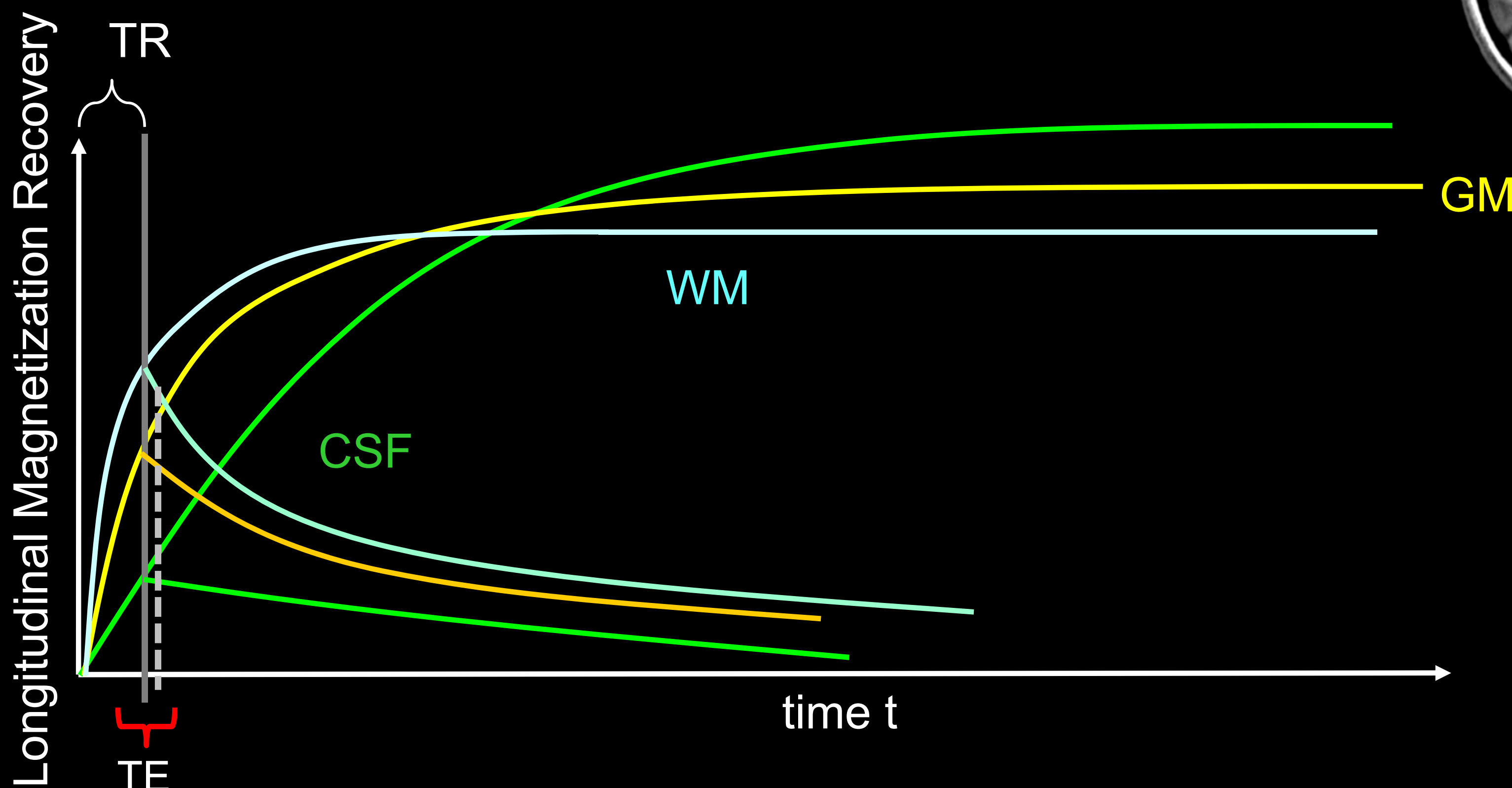


TR
GM
WM
CSF
Longitudinal Magnetization Recovery
time t
TE

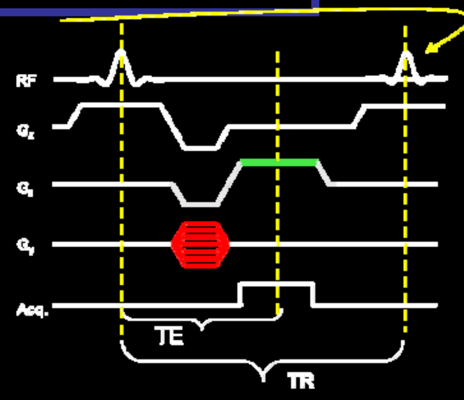
Contrast…T_1 weighting

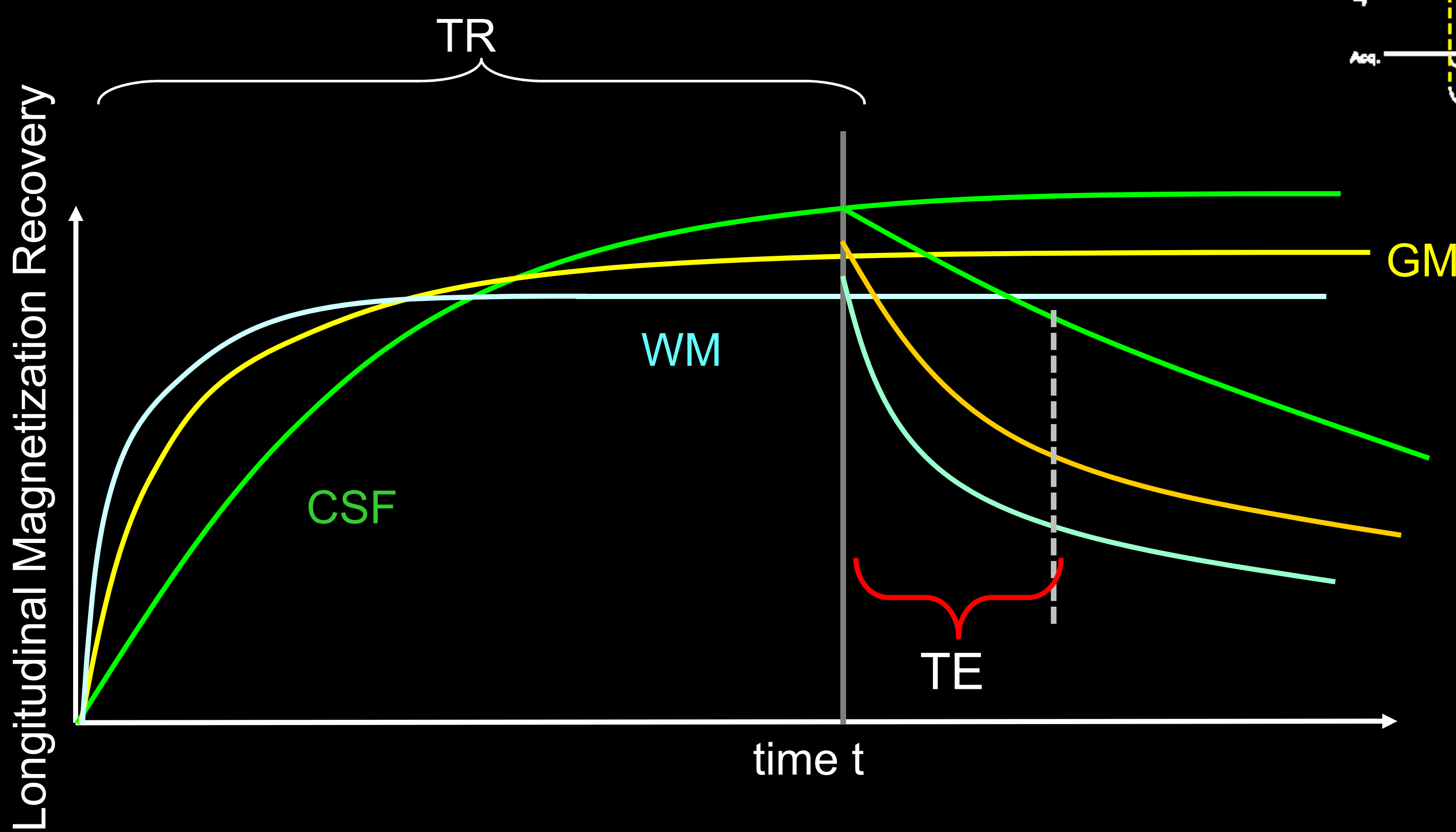
Short repetition time -TR
Short echo time - TE





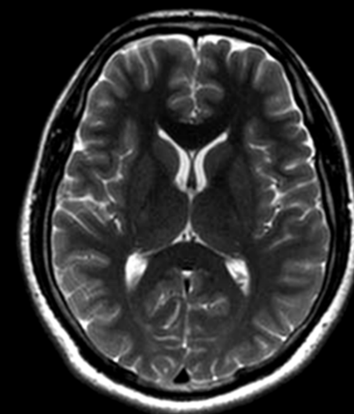
Contrast

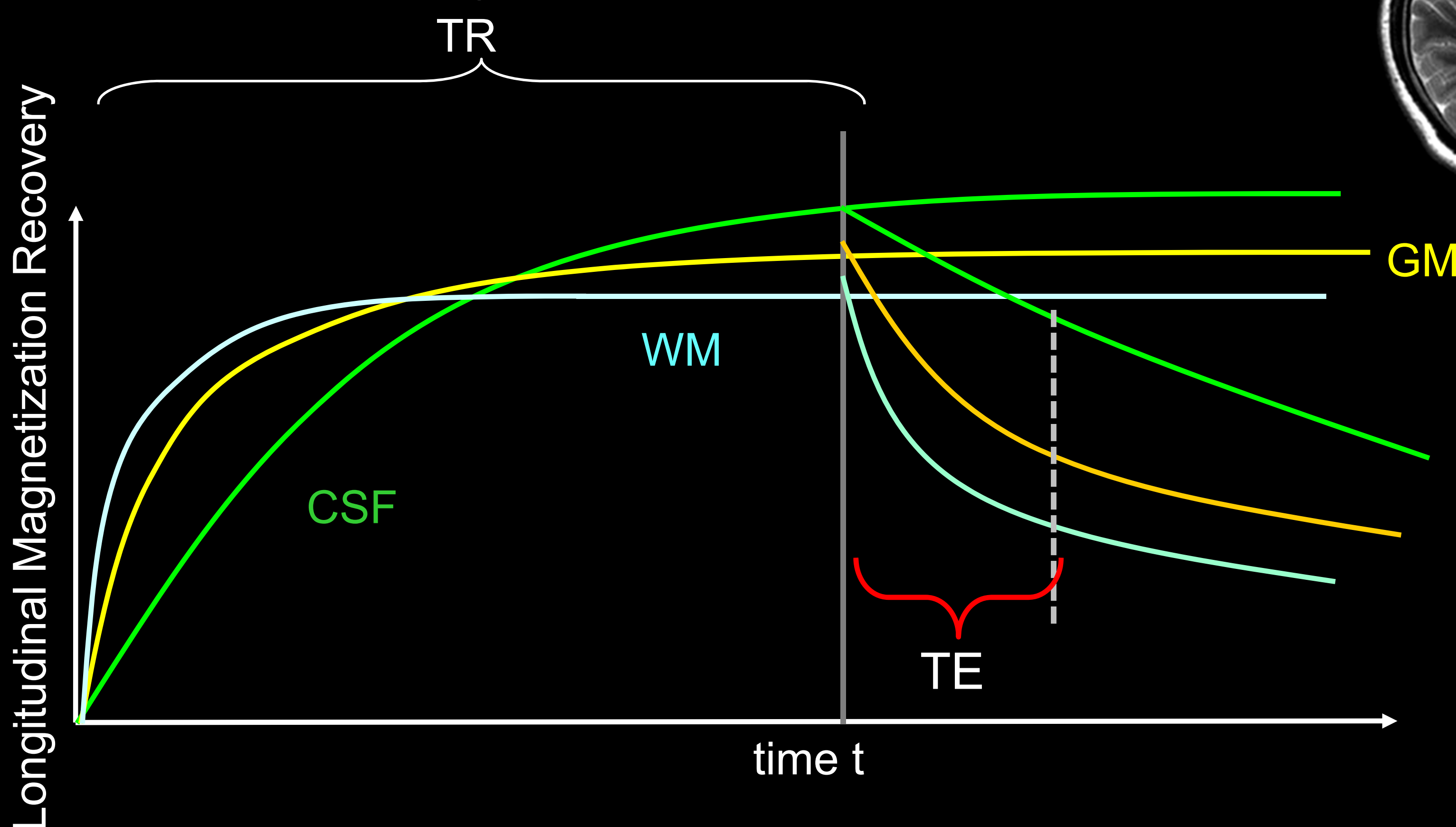

RF
Acq.
TE
TR


TR
Longitudinal Magnetization Recovery
GM
WM
CSF
TE
time t

Contrast…T_2 weighting

Long repeat time - TR
Long Echo time - TE





Most important contrast types

- ρ_0, T1 and T2 (T2*)

$$S_A(TR,TE) = \rho_{0,A}(1 - e^{-TR/T_{1,A}})e^{-TE/T^*_{2,A}}$$

$$S_B(TR,TE) = \rho_{0,B}(1 - e^{-TR/T_{1,B}})e^{-TE/T^*_{2,B}}$$

$$\Downarrow$$

$$C_{AB}(TR,TE) = S_A(TR,TE) - S_B(TR,TE)$$

Most important contrast types

$S(TR,TE) = \rho_0(1 - e^{-TR/T_1})e^{-TE/T_2^*}$

- **Spin density weighting**

TR□T1 **=>** e^{-TR/T_1} **->0**

TE□T2* **=>** e^{-TE/T_2^*} **->1**

$$C_{AB}(TR,TE) \approx \rho_{0,A} - \rho_{0,B}$$ **+□**

Note: $\rho_{0,A}$ and $\rho_{0,B}$ are effective spin density

Most important contrast types

- **T1 weighting**

$$S(TR,TE) = \rho_0(1-e^{-TR/T_1})e^{-TE/T_2^*}$$

use TE□T2* => e^{-TE/T_2^*} ->1

$$C_{AB}(TR) \approx \rho_{0,A} - \rho_{0,B} - (\rho_{0,A}e^{-\frac{TR}{T_{1,A}}} - \rho_{0,B}e^{-\frac{TR}{T_{1,B}}})$$

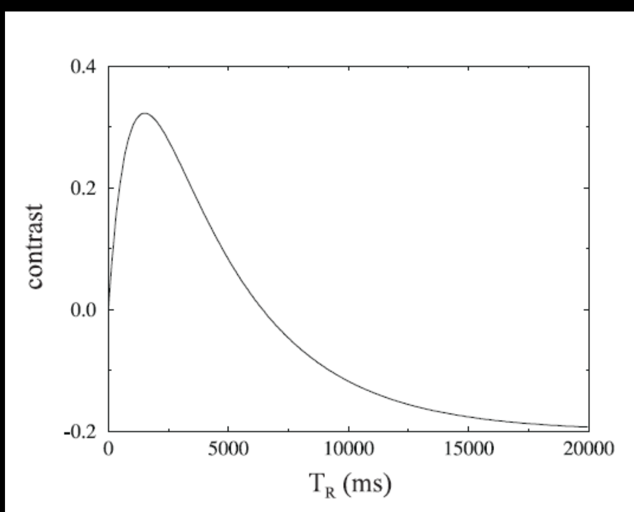


Fig. 15.7b

$$TR_{opt} = \frac{\ln\left(\frac{\rho_{0,B}}{T_{1,B}}\right) - \ln\left(\frac{\rho_{0,A}}{T_{1,A}}\right)}{\frac{1}{T_{1,B}} - \frac{1}{T_{1,A}}}$$

$$= T_{1,A}|_{T_{1,}}$$

T1 values at 3T
GM : ~1200ms
WM: ~800ms
CSF: ~2-4s

Most important contrast types

$$S(TR,TE) = \rho_0(1 - e^{-TR/T_1})e^{-TE/T_2^*}$$

- **T2* weighting**

  use TR□T1 =>

e^{-TR/T_1} **->0**

$$C_{AB}(TE\,) = \rho_{0,A}\, e^{-\frac{TE}{T_{2,A}^*}} - \rho_{0,B}\, e^{-\frac{TR}{T_{2,B}^*}}$$

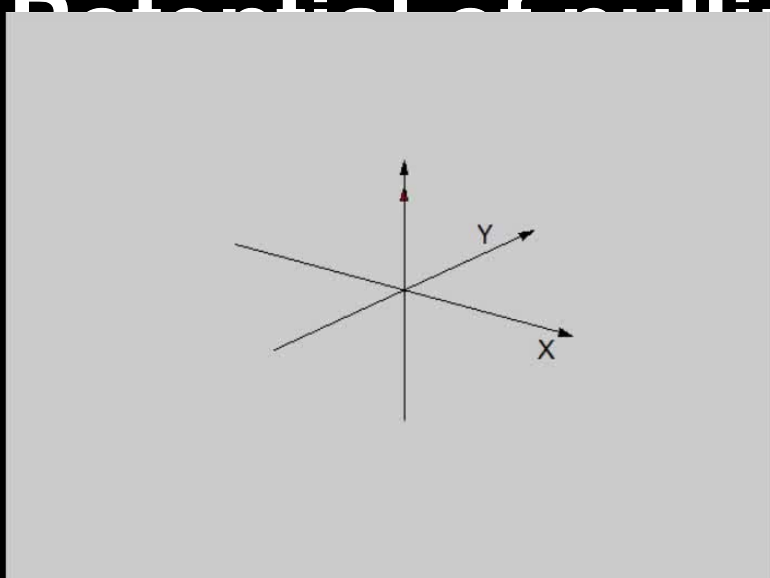
$$TE_{opt} = \frac{\ln\left(\frac{\rho_{0,B}}{T_{2,B}^*}\right) - \ln\left(\frac{\rho_{0,A}}{T_{2,A}^*}\right)}{\frac{1}{T_{2,B}^*} - \frac{1}{T_{2,A}^*}}$$

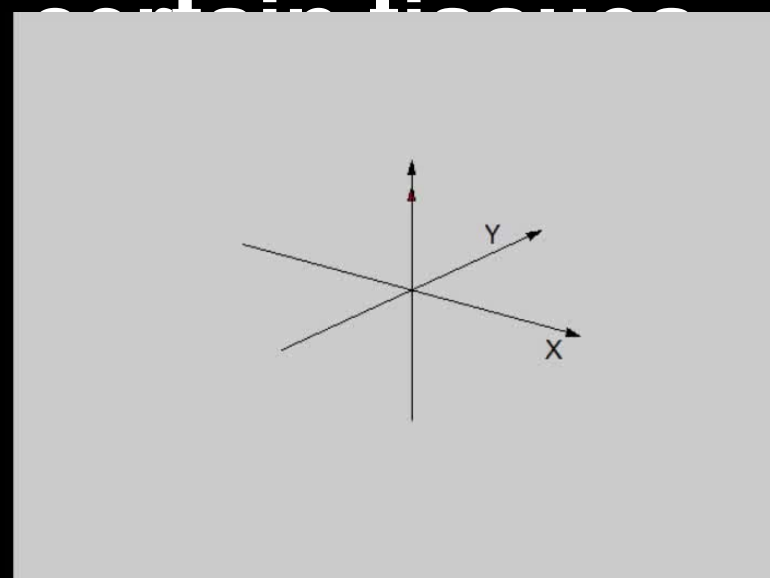
$$= T_{2,A}^*\big|_{T_{2,A}^* \cong T_{2,B}^*}$$

T1 weighting with inversion recovery (Chap 8.4)

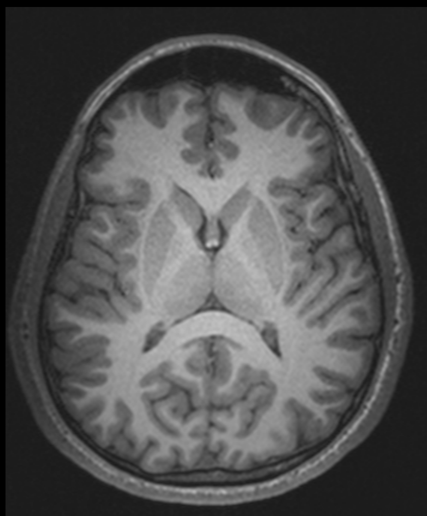
$$M_\perp(TI,TE) = M_0\left(1 - 2e^{-\frac{TI}{T_1}}\right)e^{-\frac{TE}{T_2^*}}$$

- IR may double the T1 contrast relative to GE
- Optimal TI (?)
- Potential of nulling certain tissues

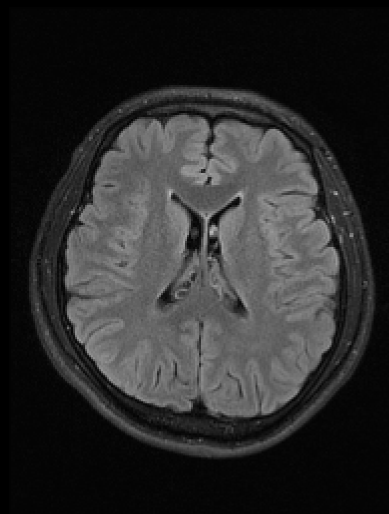




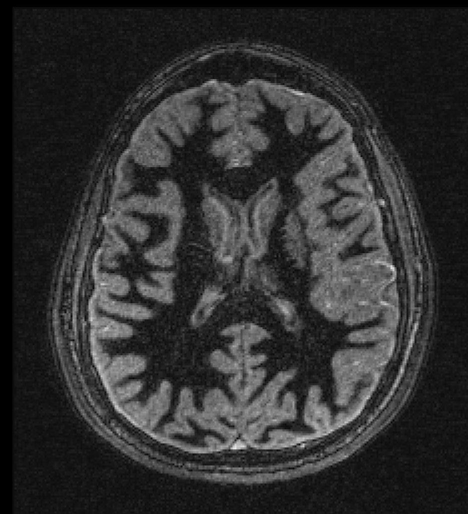
IR examples



T1W



CSF nulling, T2W



WM+CSF nulling

GM+CSF nulling

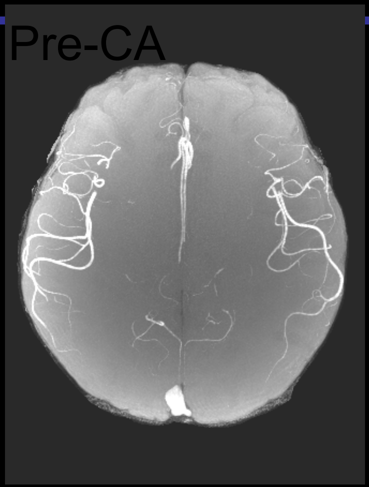
T2W

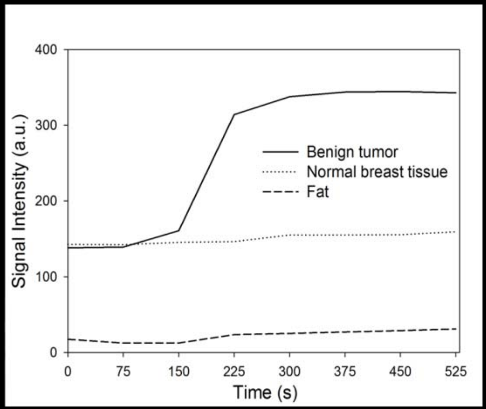
Fat nulling,T2W

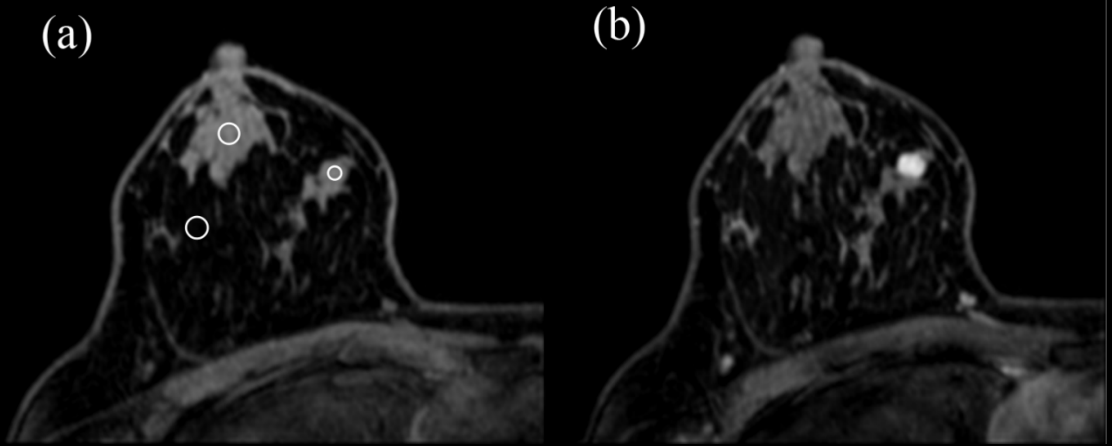
Contrast enhancement with T1-shortening agents (Gd)

- With T1 shortened , target tissue will have increased signal in short TR GE images
- T1 shortening effect proportional to agent concentration
- Target tissue example:
  - Blood (MR angiography, perfusion weighted imaging)
  - Tumor
  - Vessel wall lesion

Contrast enhanced images







CNR, partial volume and resolution

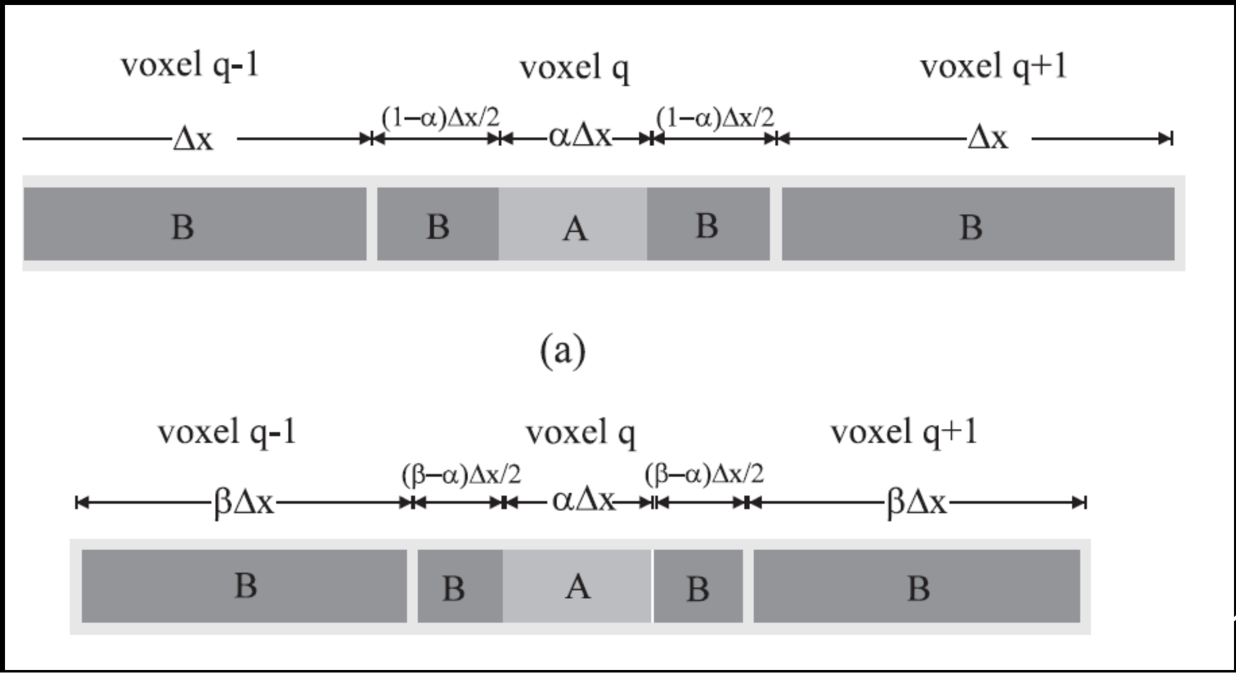
$$\mathcal{V}_{AB} \equiv \frac{C_{AB}}{\sigma_{eff}} = \frac{C_{AB}}{\sigma_0}\sqrt{N_{acq}} = \frac{C_{AB}}{\sigma_0}\sqrt{n_{voxel}}$$

- **Resolution □ -> σ_0 □ -> n_{voxel} □ -> $\mathcal{V}_{AB}$(?)**
- **Partial volumed voxel contains >1 tissues/objects**
- **Partial volume effect (PVE) is inevitable in in-vivo MR imaging**
  - **Finite voxel size (0.25mm ~ 4mm)**
  - **Complex tissue boundaries**
  - **Micro vasculature/structures**
  - **…**

CNR, partial volume and resolution

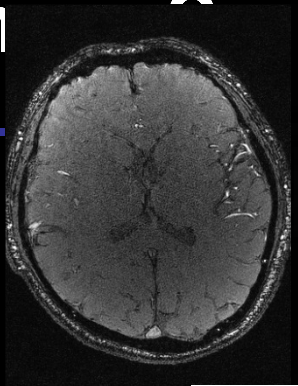
- **Size of A: $\alpha\Delta x$, (α<1)**
- **Size of B: Δx or $\beta\Delta x$, (β<1)**
- **In both scenarios: $C_{AB} = \alpha(S_A - S_B)$**
- **Noise σ:** $\sigma(\alpha) = \sigma_0$, **($\alpha L_x$, N)**

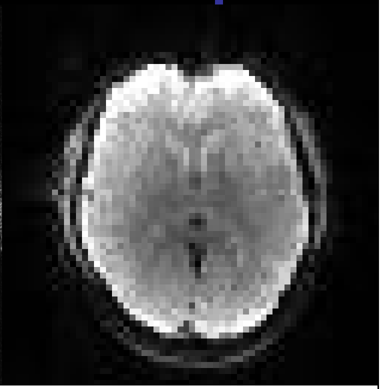
  $\sigma(\alpha) = \sigma_0/\sqrt{\alpha}$, **($L_x$, αN)**

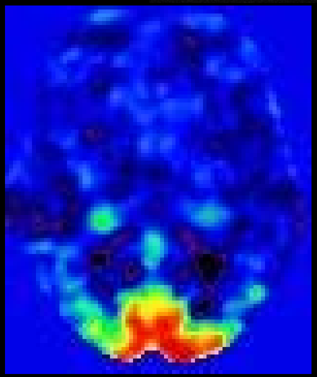


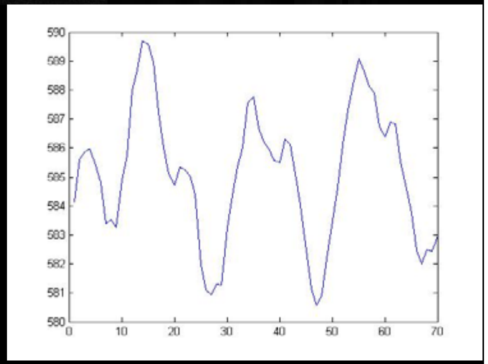
How PVE affects the im

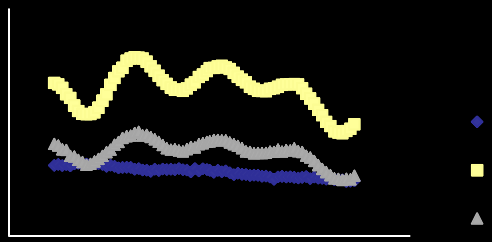
- Blurring
- Signal dephase (T2'/T2*, BOLD imaging, chap 25)
- Double/multiple T2/T2* exponential decay
- Signal cancellation (water/fat)



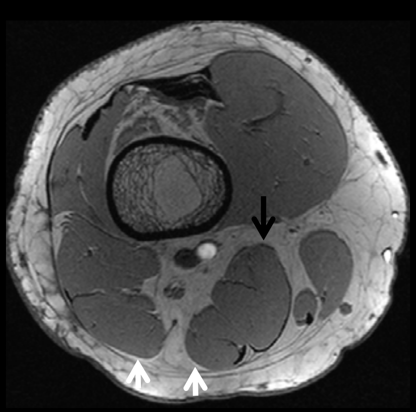


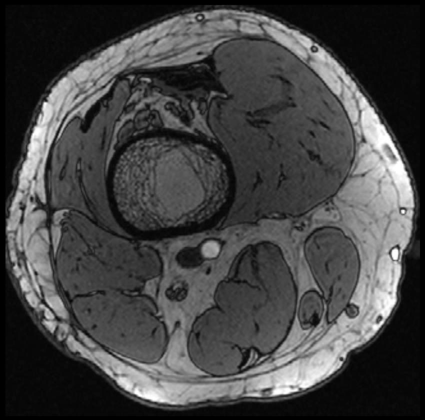






Note the similarity with Fig. 17.8
Courtesy of Manju





Signal and noise of phase images

- S_{phase} is affected by chemical shift, local susceptibility and/or motion, offering different contrast than magnitude images
- S_{phase} is a relative value since baseline $\phi_0$is unknown
- Only the contrast in phase has useful information
- $\sigma_{phase} = 1/SNR_{mag}$
- Phase may still provides sufficient contrast when magnitude fails

Phase imaging applications

- Chemical shift (fat/water) imaging (Chap. 17)
- B_0 field homogeneity mapping (Chap. 20)
- Phase contrast MR angiography (Chap. 24)
- Blood flow quantification (Chap. 24)
- Susceptibility Weighted Imaging (SWI, Chap. 25)
- …

Homework

- Prob 15.1, 15.3, 15.4, 15.8, 15.10, 15.12, 15.13

Next Session

Chapter 15.3-15.6

Most important contrast types

T1 values at 3T
GM : ~1200ms
WM: ~800ms
CSF: ~2-4s

- **Examples:**