Chapter 15 – Signal, Contrast and Noise

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Signal and noise

- **Noise in k-space**
  - **Thermal noise (body/object, coil, electronics)**
  - **Systematic noise (scanner, AC power, etc)**
  - **Physiological noise (heartbeat, respiration, etc)**

- **Thermal noise estimation**

$$\sigma^2_{thermal} = \sigma^2_{body} + \sigma^2_{coil} + \sigma^2_{electronics}$$
$$\propto 4kT \cdot R \cdot BW_{readout}$$

Noise

$$\boldsymbol{s_m(k) = s(k) + \epsilon(k)}$$

- Noise in MR image

$$\eta(p\Delta x) = iDFT[\epsilon(k)] = \frac{1}{N}\sum_{p'} \epsilon(p'\Delta k)e^{i2\pi p'\Delta k p\Delta x}$$

- Properties of noise in image

$$mean\big(\eta(p\Delta x)\big) = iDFT\left[\overline{\epsilon(k)}\right] = 0$$

$$var\big(\eta(p\Delta x)\big) \equiv DFT[R_\epsilon(\tau)] = \frac{\sigma_m^2}{N}$$

Noise

$$\boldsymbol{s_m(k) = s(k) + \epsilon(k)}$$

$$s_{m,r}(k) = R(k) + \epsilon_r(k) \qquad s_{m,I}(k) = I(k) + \epsilon_I(k)$$

Magnitude image $\quad S_{mag} = \sqrt{I^2 + R^2}$

A function of random variables

$$M = f(u, v, \dots)$$

$$\sigma_M^2 \simeq \sigma_u^2 \left(\frac{\partial M}{\partial u}\right)^2 + \sigma_v^2 \left(\frac{\partial M}{\partial v}\right)^2 + \cdots + 2\sigma_{uv}^2 \left(\frac{\partial M}{\partial u}\right)\left(\frac{\partial M}{\partial v}\right)$$

Using this error propagation equation…

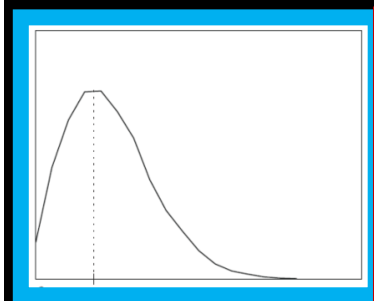
$$\boxed{\sigma_{S_{mag}} = \sigma_0}$$

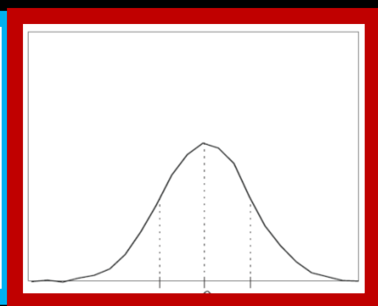
Assuming noise in real and imaginary channels is uncorrelated and equal

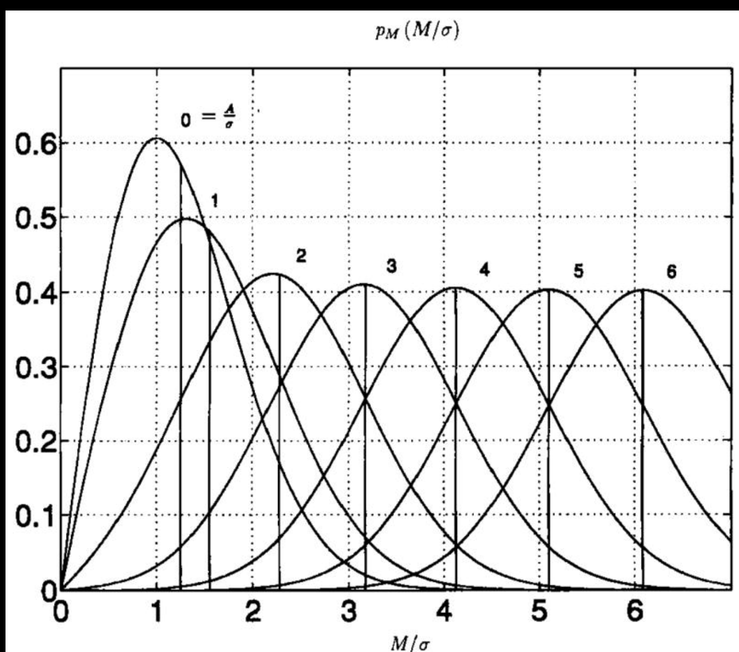
$$\sigma_0 = \sigma_I = \sigma_R$$

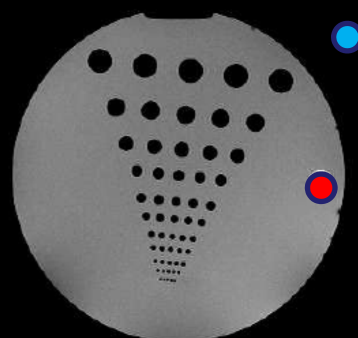
Measuring SNR

(σ_{bg} = 0.655 σ_0, Rayleigh distribution), $\sigma_{oj} > \sigma_{bg}$

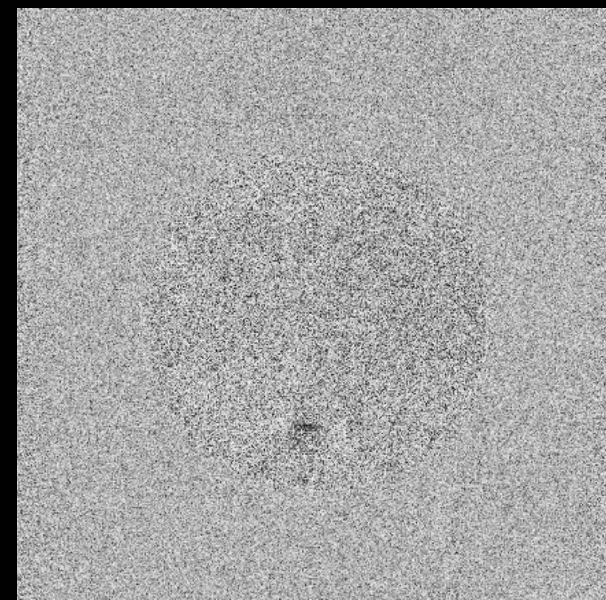








Single Acq



subtracted

$A/\sigma \geq 3$, gaussian distribution

Gudbjartsson H, Patz S. The Rician distribution of noisy MRI data. Magn Reson Med. 1995 Dec;34(6):910-4.

Noise in phase domain

$$s_{m,r}(k) = R(k) + \epsilon_r(k) \qquad s_{m,I}(k) = I(k) + \epsilon_I(k)$$

Phase image $\phi = \tan^{-1}(I/R)$

$$M = f(u, v, \dots)$$

$$\sigma_M^2 \simeq \sigma_u^2 \left(\frac{\partial M}{\partial u}\right)^2 + \sigma_v^2 \left(\frac{\partial M}{\partial v}\right)^2 + \cdots + 2\sigma_{uv}^2 \left(\frac{\partial M}{\partial u}\right)\left(\frac{\partial M}{\partial v}\right)$$

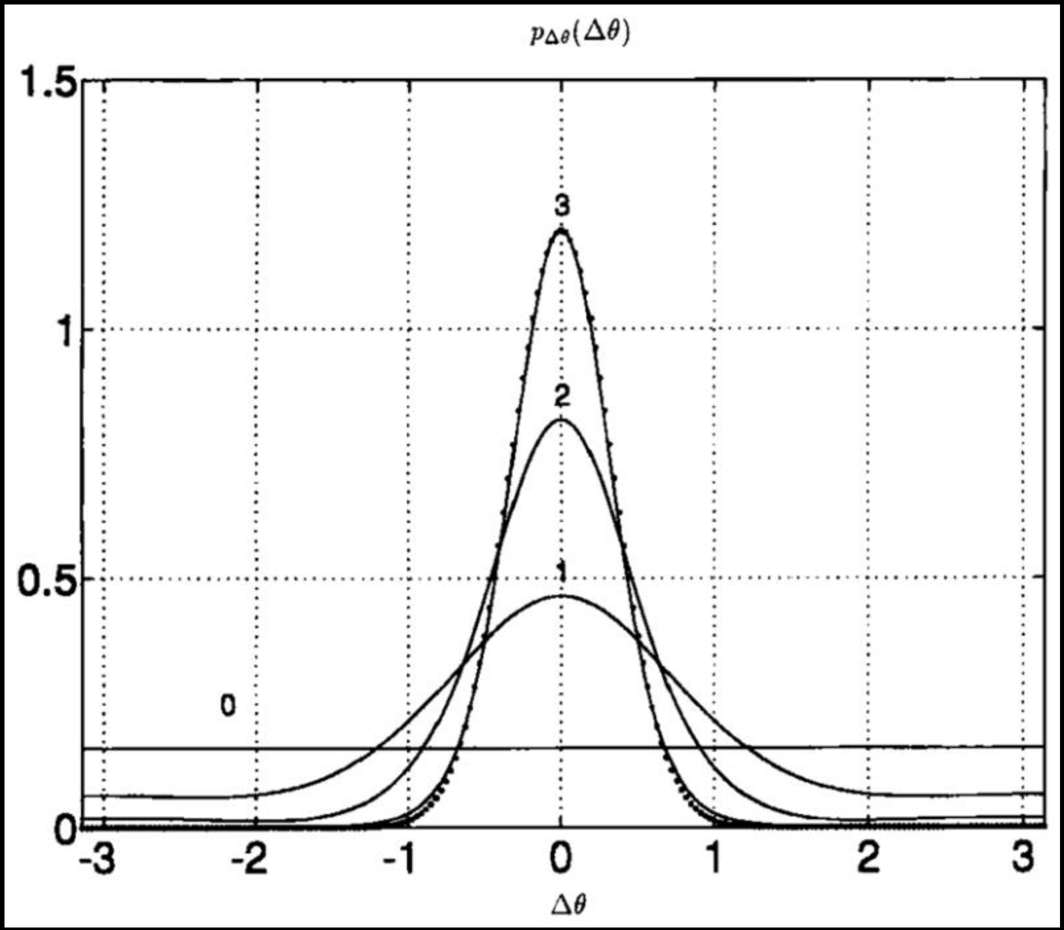
Using this error propagation equation…

$$\boxed{\sigma_\phi = \sigma_0 / S_{mag}}$$

Assuming noise in real and imaginary channels is uncorrelated and equal

$$\sigma_0 = \sigma_I = \sigma_R$$

Noise in phase domain



Distribution of phase noise as a function of A/σ

$$\sigma_\phi = \frac{\sigma_0}{S_{mag}} = \frac{1}{SNR_{mag}}$$

SNR dependence on imaging parameters

Voxel signal $\propto \Delta x$ (Voxel volume)

Noise standard deviation $\sigma \propto \frac{1}{\sqrt{N_x}}$

Noise standard deviation $\sigma \propto \frac{1}{\sqrt{N_{acq}}}$

Noise standard deviation $\sigma \propto \sqrt{BW_{readout}}$

$$SNR/voxel \equiv \frac{Signal_{voxel}}{\sigma} \propto \frac{\Delta x \Delta y \Delta z \sqrt{N_{acq}}}{\sqrt{\frac{BW_{read}}{N_x N_y N_z}}}$$

SNR dependence on imaging parameters

$$SNR/voxel \propto \frac{\Delta x \Delta y \Delta z \sqrt{N_{acq}}}{\sqrt{\frac{BW_{read}}{N_x N_y N_z}}}$$

$$\frac{1}{\Delta t} = \gamma \cdot G_x \cdot N_x \cdot \Delta x$$ Total read bandwidth

$$\frac{SNR}{voxel} \propto \Delta x \Delta y \Delta z \sqrt{N_{acq} N_x N_y N_z \Delta t}$$

$$T_s = N_x \Delta t$$

$$\frac{SNR}{voxel} \propto \Delta x \Delta y \Delta z \sqrt{N_{acq} N_y N_z T_s}$$

$$(\mathrm{SNR}/\mathrm{voxel})|_{2D} \propto \Delta x \Delta y T H \sqrt{N_y T_s}$$

Contrast, CNR and visibility

- **Contrast: simply the signal difference**

$$C_{AB} \equiv S_A - S_B$$

- **CNR: contrast-to-noise ratio**

$$CNR_{AB} \equiv SNR_A - SNR_B$$

when $\sigma_A = \sigma_B = \sigma_0$,

$$CNR_{AB} = \frac{C_{AB}}{\sigma_0}$$

Contrast, CNR and visibility

- **Visibility**

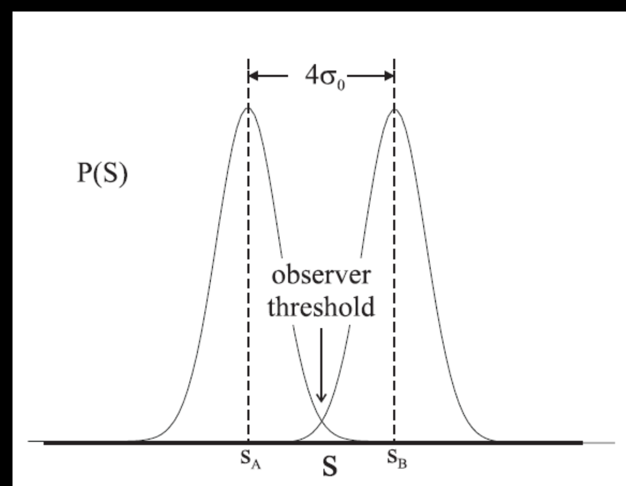
$$\mathcal{V}_{AB} \equiv \frac{C_{AB}}{\sigma_{eff}} = \frac{C_{AB}}{\sigma_0}\sqrt{N_{acq}} = \frac{C_{AB}}{\sigma_0}\sqrt{n_{voxel}}$$

Temporal

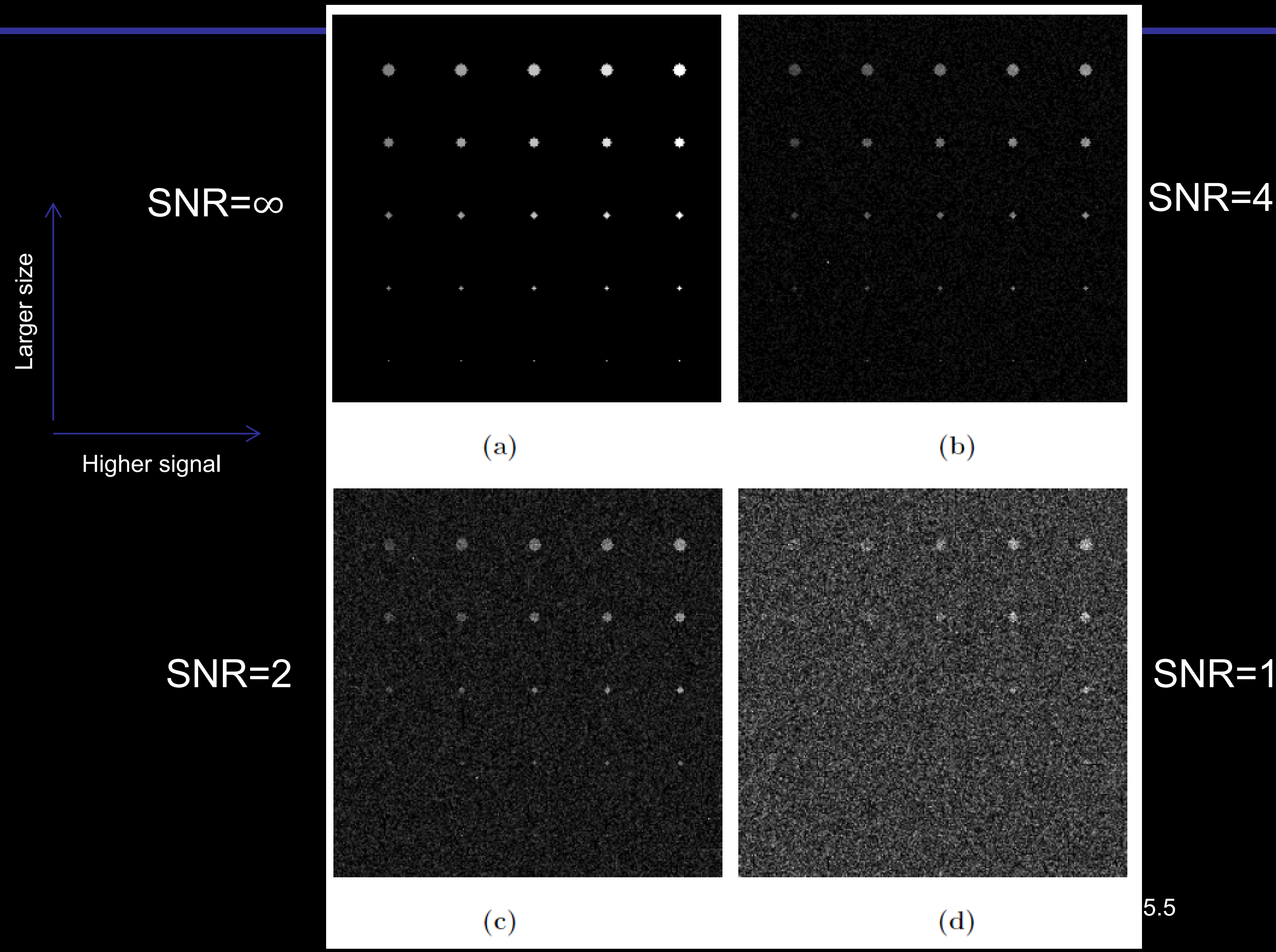
Spatial

- **The Rose Criterion**

$$CNR_{AB} > 3\sim5$$



Contrast, CNR and visibility



This class - Key points

- Noise (signal to noise ratio) -> relation to experimental parameters
- SNR in magnitude and phase domain
- Contrast, its optimization
- Partial voluming
- Affect of common exogenous contrast agents

Contrast mechanisms in MRI

- **MRI is a multiple contrast imaging modality**

Intrinsic:
- spin density
- T1/T2/T2*/T1ρ
- Diffusion
- Chemical shift
- Susceptibility
- Temperature
- …

Physiological:
- Blood flow
- Perfusion
- Blood oxygenation
- Tissue elasticity
- …

Sequence specific:
- Steady state (chap.18)
- Selective excitation /suppression
- …

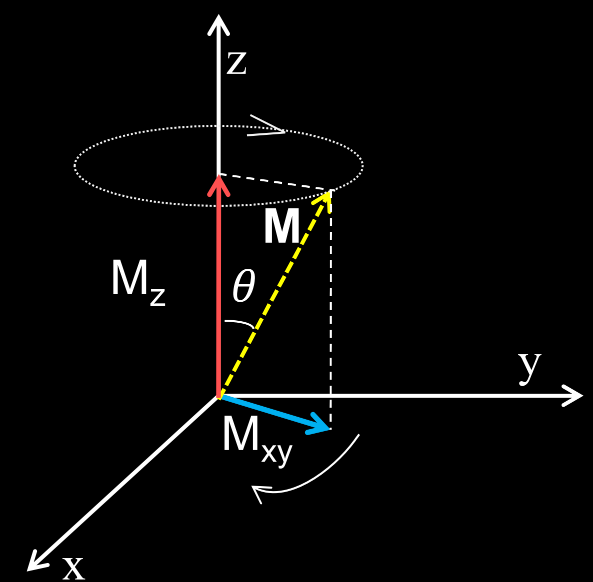
- **Or multiple methods for the same contrast**
  - **Example:**
    T1W : short TE/TR GE, or IR
    Signal nulling: IR, selective excitation/saturation, dephasing

Basic Contrast Mechanisms: Relaxation effects

$$\frac{d\vec{M}}{dt} = \gamma \cdot \vec{M} \times B + \frac{M_0 - M_z}{T_1} - \frac{M_{xy}}{T_2}$$

M_0 is the equilibrium magnetization

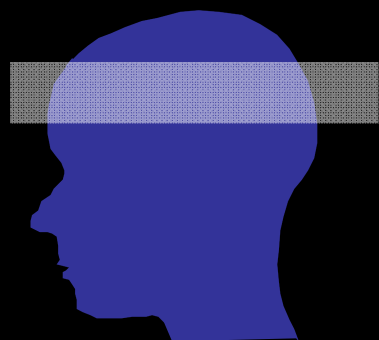
M_{xy} is the transverse magnetization

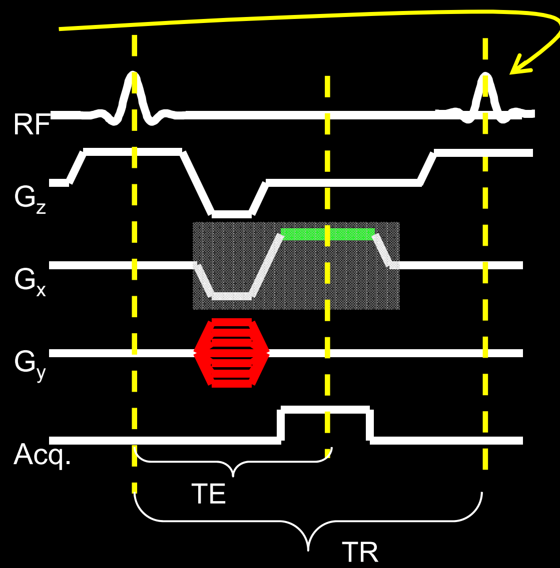


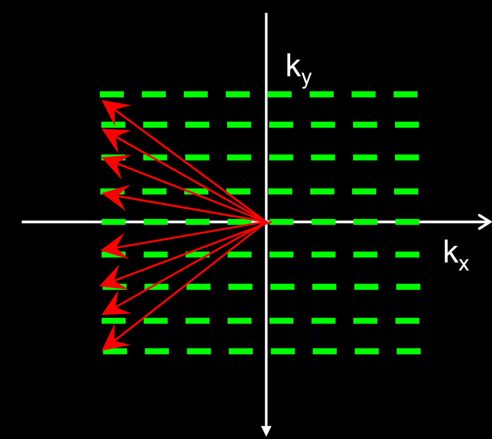
Contrast

Tissue	T_1 (ms)	T_2 (ms)
gray matter (GM)	950	100
white matter (WM)	600	80
muscle	900	50
cerebrospinal fluid (CSF)	4500	2200
fat	250	60
blood[3]	1200	100-200[4]

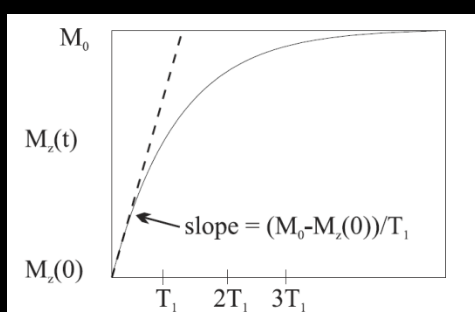
Contrast





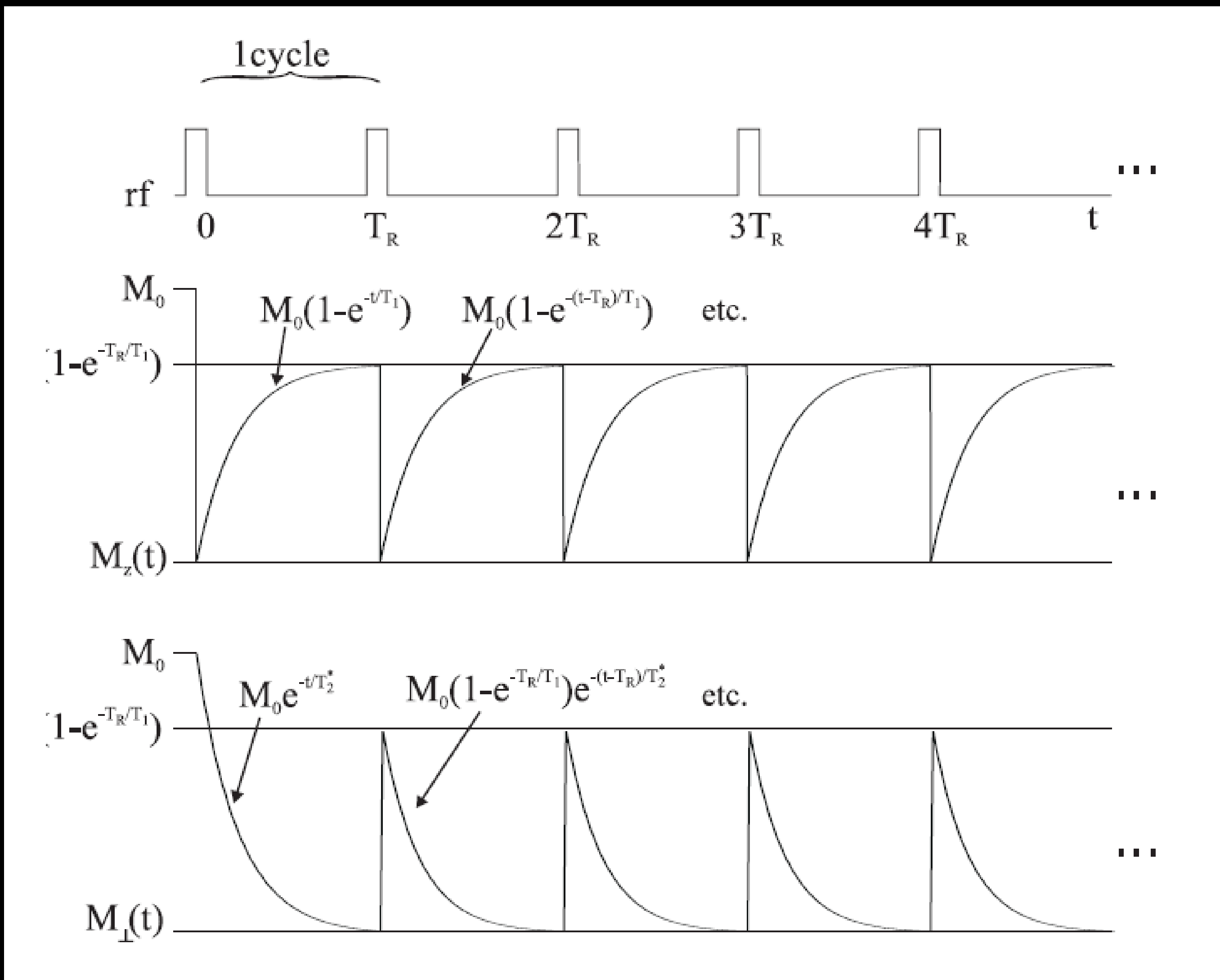


Once you tip the magnetization of a tissue, it takes a finite amount of time for it to recover

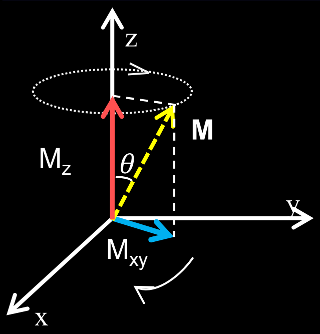


With repeated rf pulse excitations during image acquisition, the time intervals of repetition and acquisition (TR,TE), along with T1 and T2 values of different tissue determine the final contrast

Magnetization equations



Magnetization equations



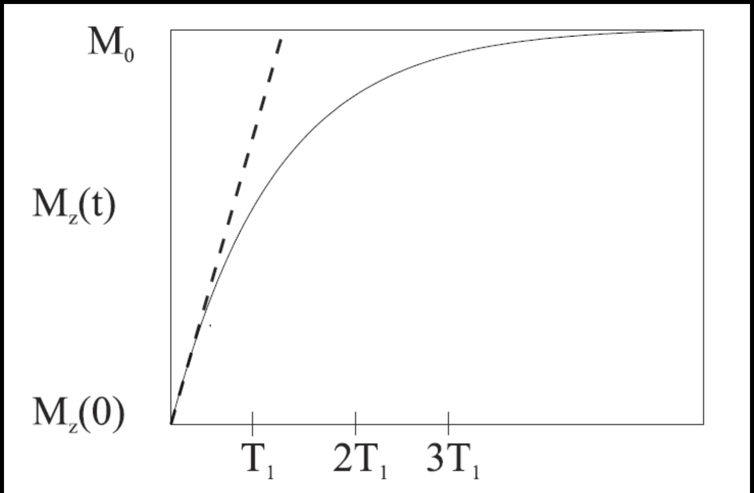
$$M_z(t) = M_z(0) \cdot e^{-\frac{t}{T_1}} + M_0 \cdot \left(1 - e^{-\frac{t}{T_1}}\right)$$

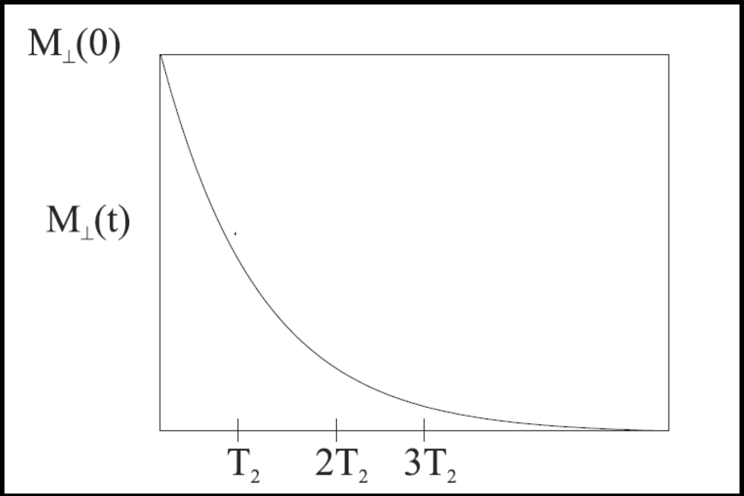
$$M_z(0) = M_0 \cdot \cos(\theta)$$

$$M_{xy}(t) = M_{xy}(0) \cdot e^{-\frac{t}{T_2}}$$

$$M_{xy}(0) = M_0 \cdot \sin(\theta)$$

$$\boxed{\theta = 90^o}$$





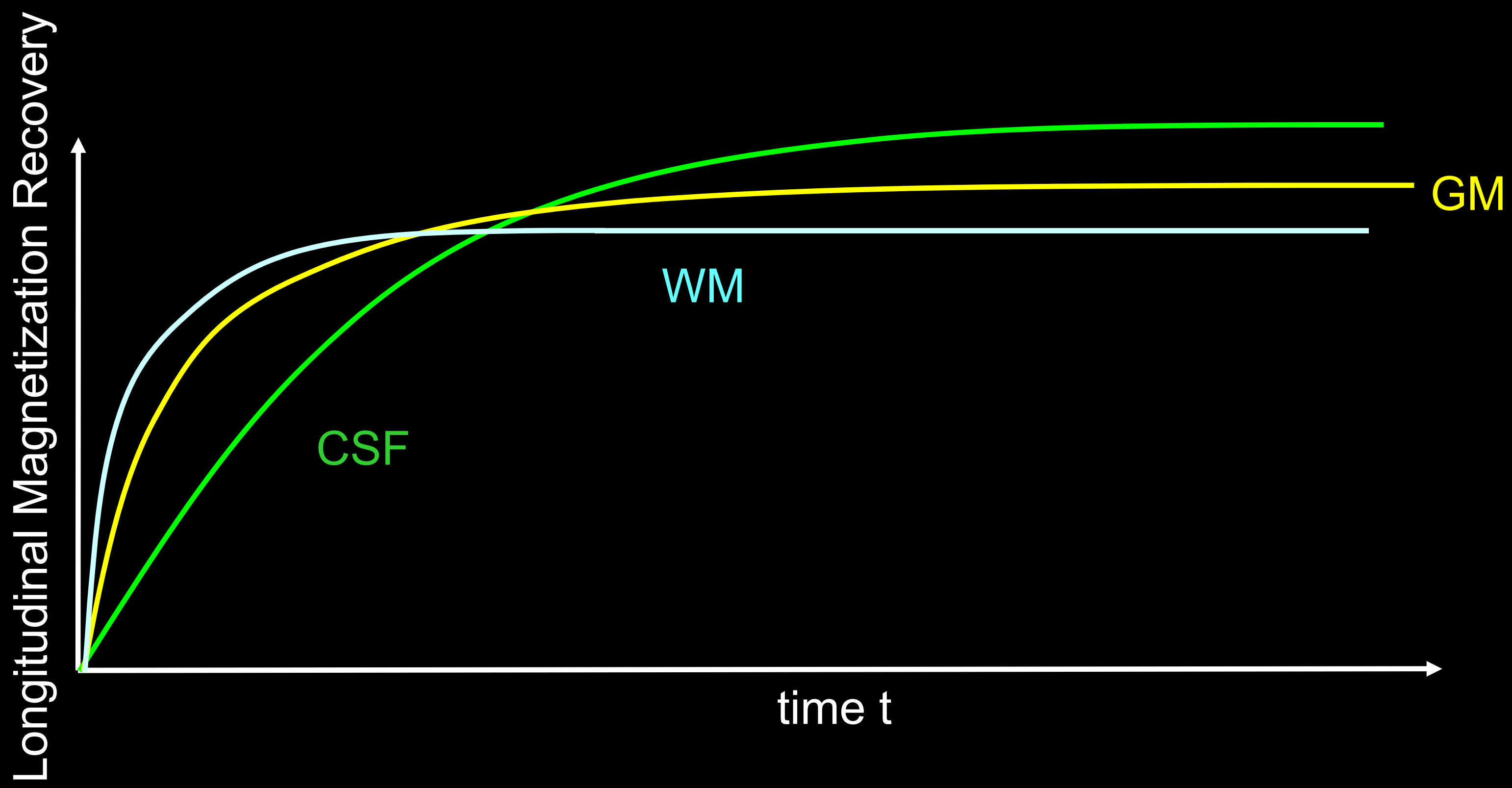
$$M_z(TR^-) = M_0 \cdot \left(1 - e^{-TR/T_1}\right)$$

$$M_{xy}(TE) = M_0 \cdot \left(1 - e^{-TR/T_1}\right) \cdot e^{-\frac{TE}{T_2}}$$

$$M_{xy}(t) = M_z(TR^-) \cdot e^{-\frac{t}{T_2}}$$

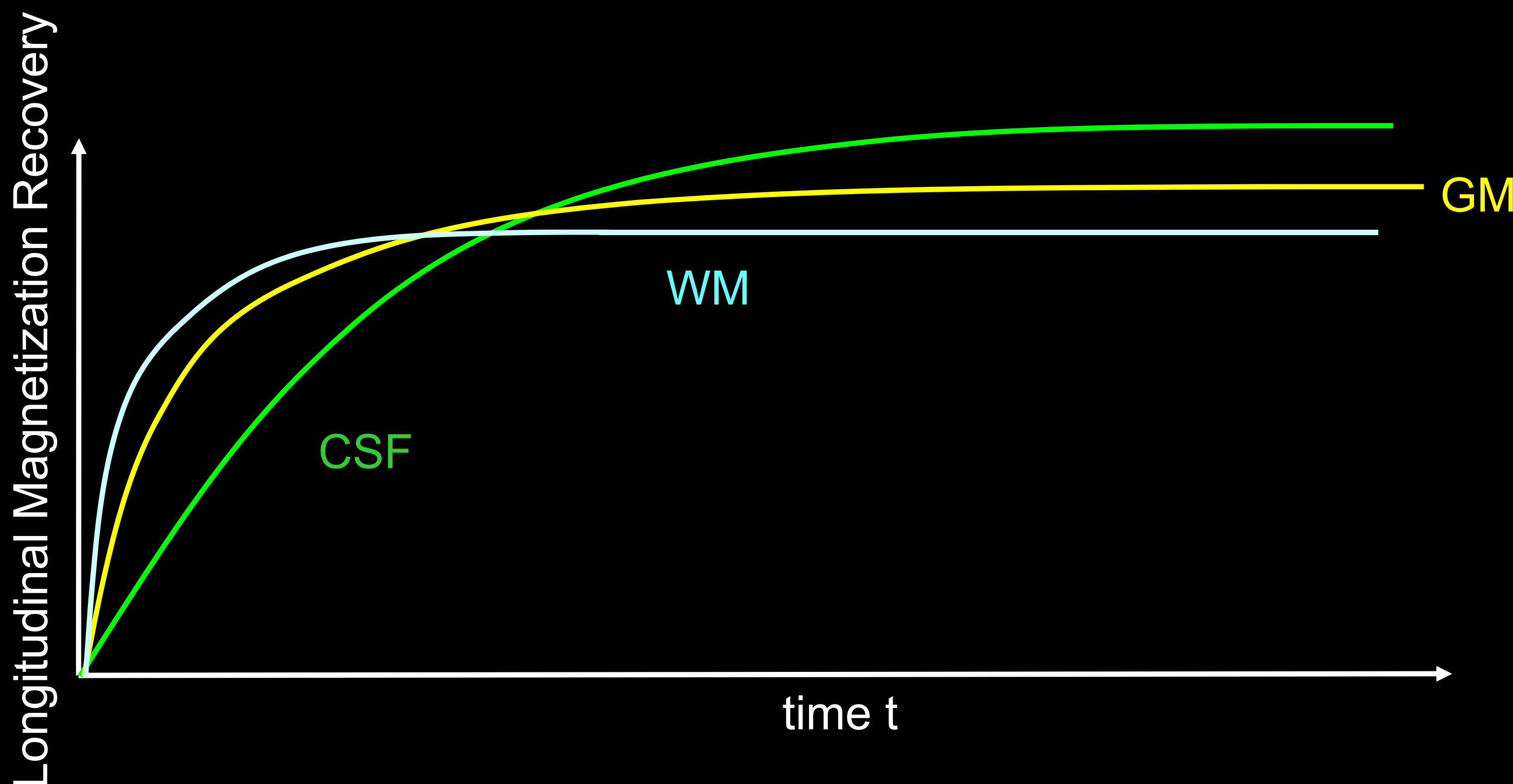
$$TR \gg T_2$$

Contrast

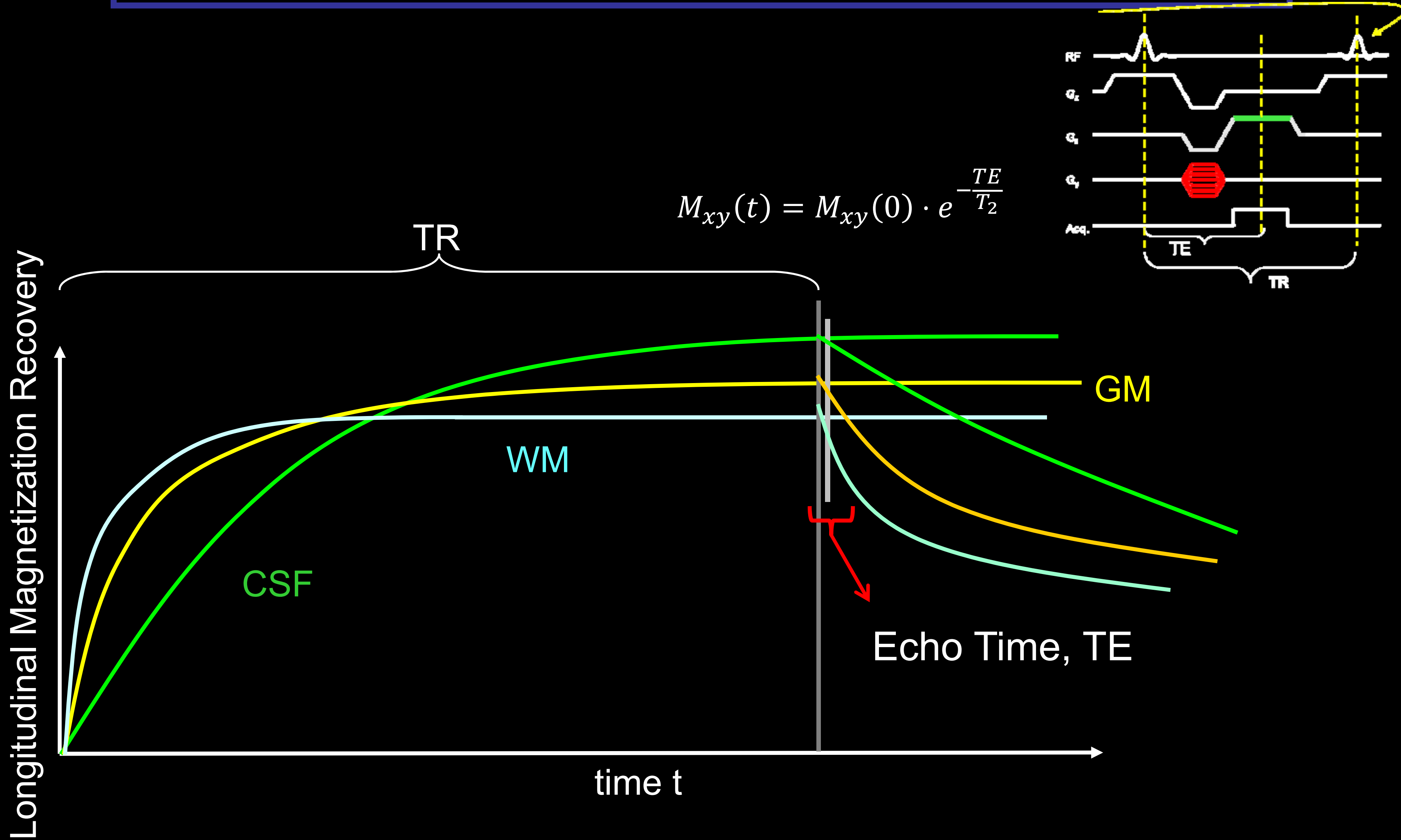


Contrast

Different tissues have different amounts of water and hence different densities of ^{1}H protons within them.

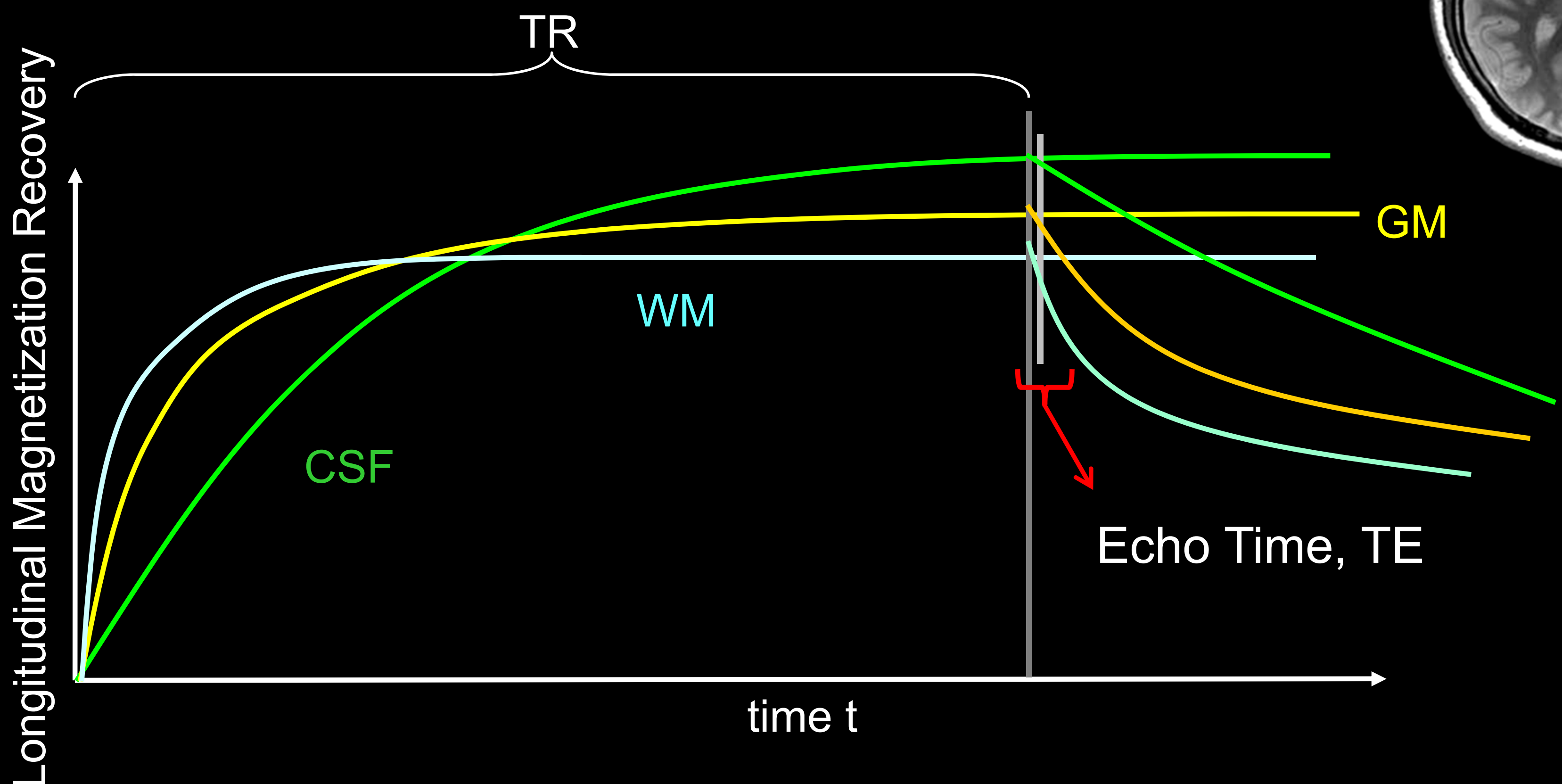


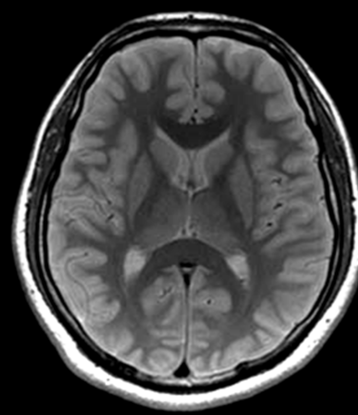
Contrast



$M_{xy}(t) = M_{xy}(0) \cdot e^{-\frac{TE}{T_2}}$
RF
Acq.
TE
TR
TR
Longitudinal Magnetization Recovery
GM
WM
CSF
Echo Time, TE
time t

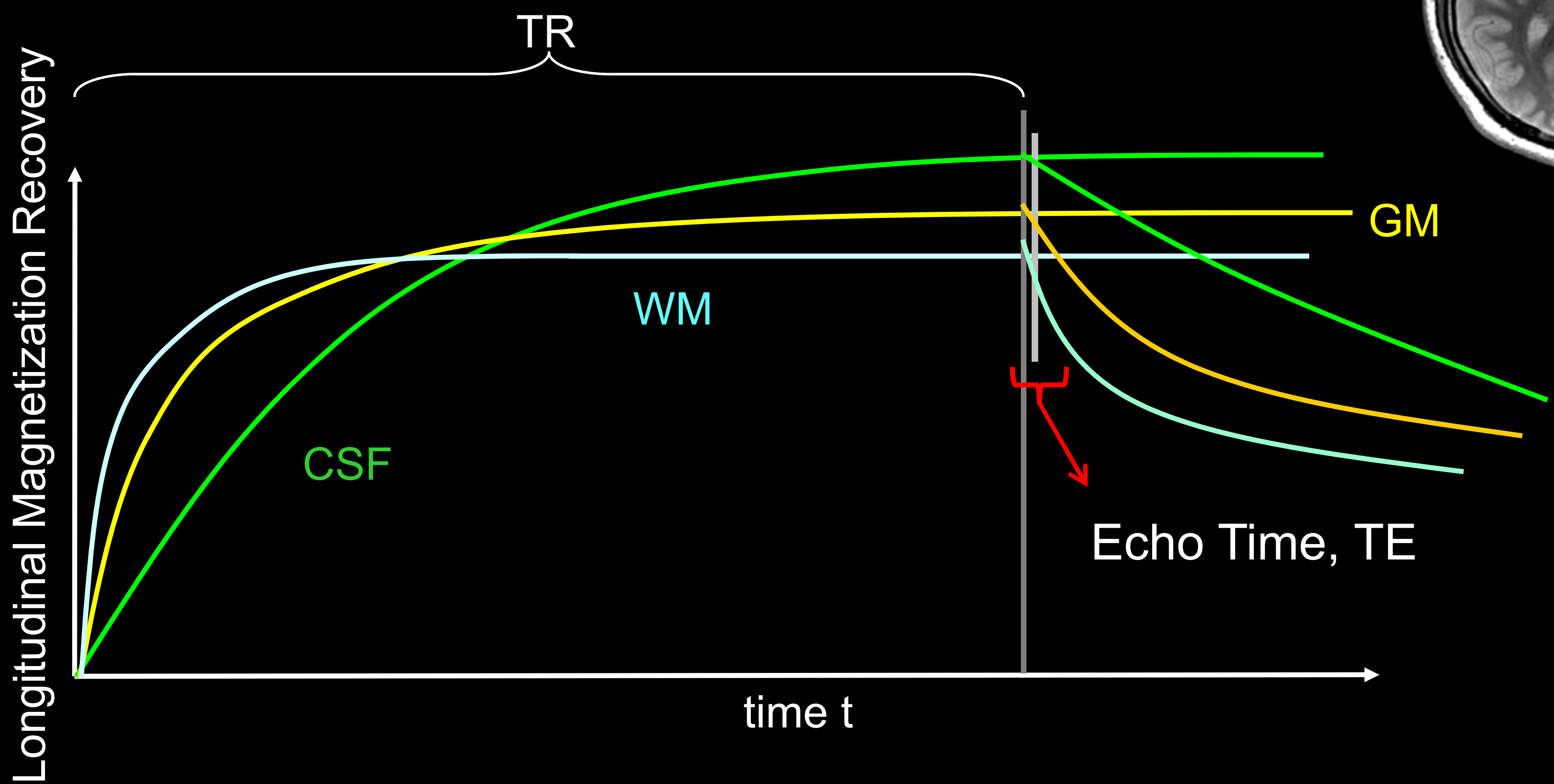
Contrast… ρ_0 weighting

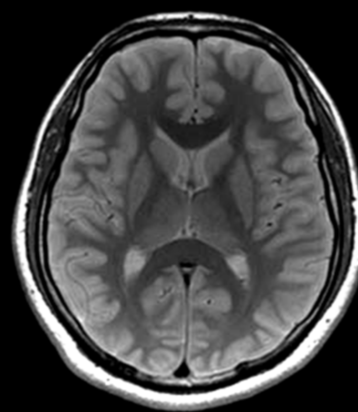




Contrast…ρ_0 weighting

Long repetition time -TR
Short echo time - TE





Contrast weighting

ρ_0, T1 and T2 (T2*)

$$S_A(TR,TE) = \rho_{0,A}(1 - e^{-TR/T_{1,A}})e^{-TE/T^*_{2,A}}$$

$$S_B(TR,TE) = \rho_{0,B}(1 - e^{-TR/T_{1,B}})e^{-TE/T^*_{2,B}}$$

$$\Downarrow$$

$$C_{AB}(TR,TE) = S_A(TR,TE) - S_B(TR,TE)$$

Spin density weighting

$S(TR, TE) = \rho_0(1 - e^{-TR/T_1})e^{-TE/T_2^*}$

$C_{AB}(TR, TE) = S_A(TR, TE) - S_B(TR, TE)$

- Spin density weighting

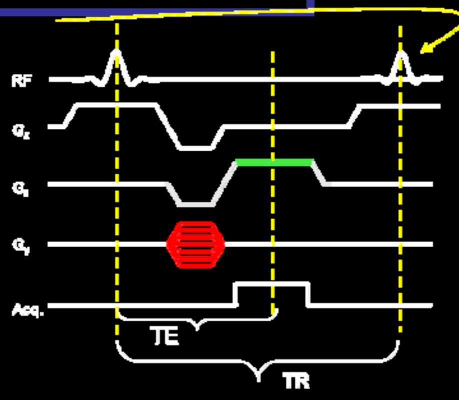
TR≫T1 => e^{-TR/T_1} ->0

TE≪T2* => e^{-TE/T_2^*} ->1

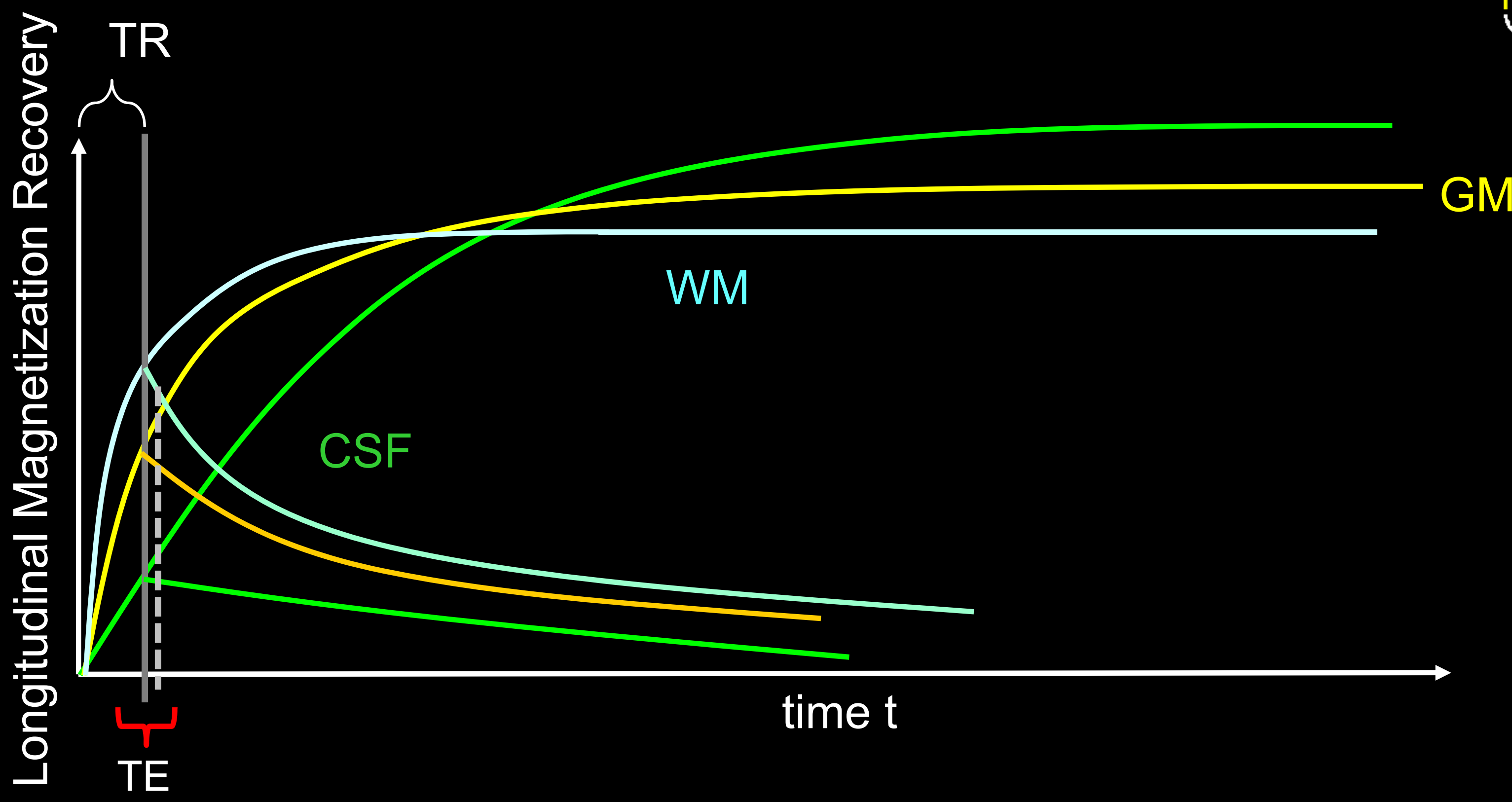
$C_{AB}(TR, TE) \approx \rho_{0,A} - \rho_{0,B}$+O()

Note: $\rho_{0,A}$ and $\rho_{0,B}$ are effective spin density

Contrast



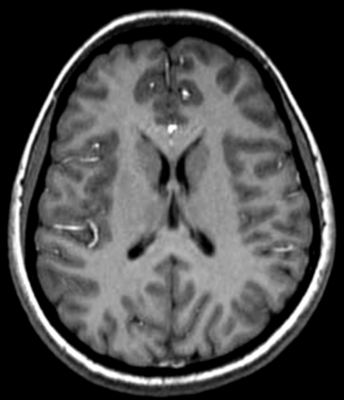
RF
Acq.
TE
TR

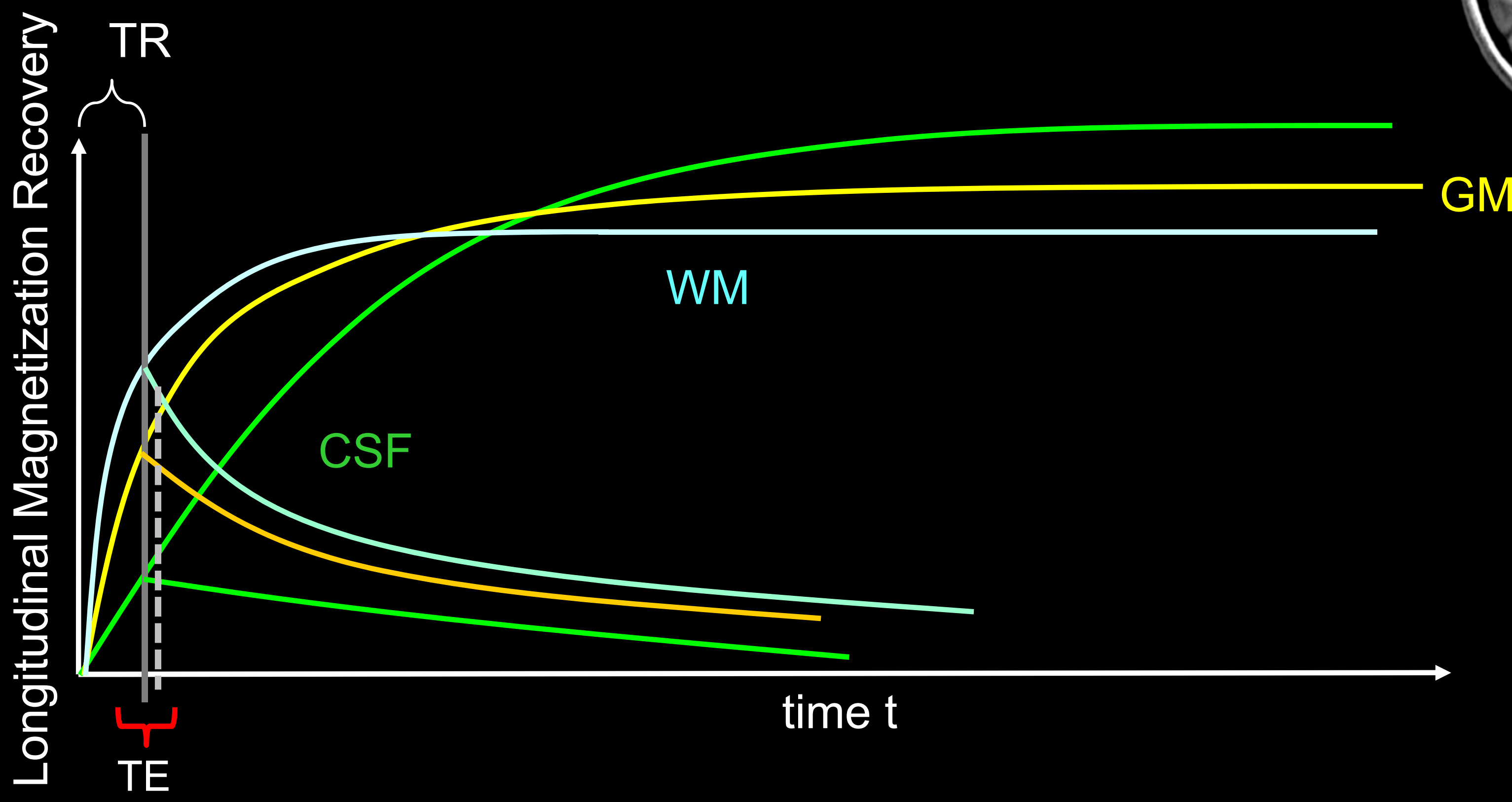


TR
Longitudinal Magnetization Recovery
GM
WM
CSF
time t
TE

Contrast... T_1 weighting

Short repetition time -TR
Short echo time - TE





T1 weighting

$$S(TR,TE) = \rho_0(1 - e^{-TR/T_1})e^{-TE/T_2^*}$$

$$C_{AB}(TR,TE) = S_A(TR,TE) - S_B(TR,TE)$$

Short TR, Short TE

$$TR \ll T_1, TE \ll T_2(T_2^*)$$

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots$$

$$S(TR,TE) \simeq \rho_0 \left(\frac{TR}{T_1}\right)$$

T1 weighting

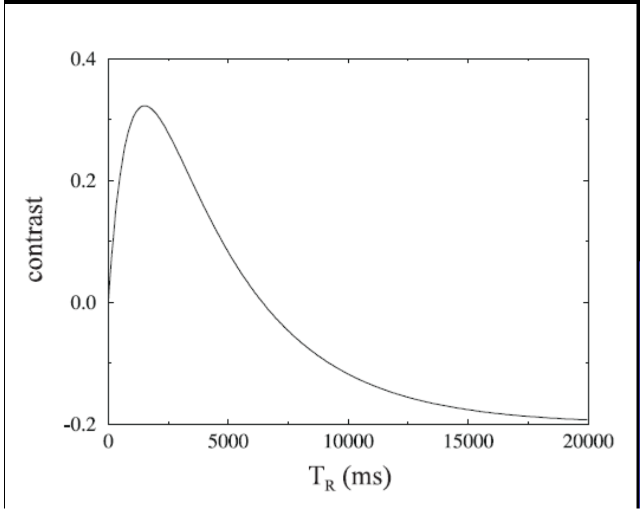
$$S(TR,TE) = \rho_0(1 - e^{-TR/T_1})e^{-TE/T_2^*}$$

Short TR, Short TE

- **T1 weighting**

  use TE≪T2* $\Rightarrow$ $e^{-TE/T_2^*} \rightarrow$ **1**

$$C_{AB}(TR) \approx \rho_{0,A} - \rho_{0,B} - (\rho_{0,A}e^{-\frac{TR}{T_{1,A}}} - \rho_{0,B}e^{-\frac{TR}{T_{1,B}}})$$

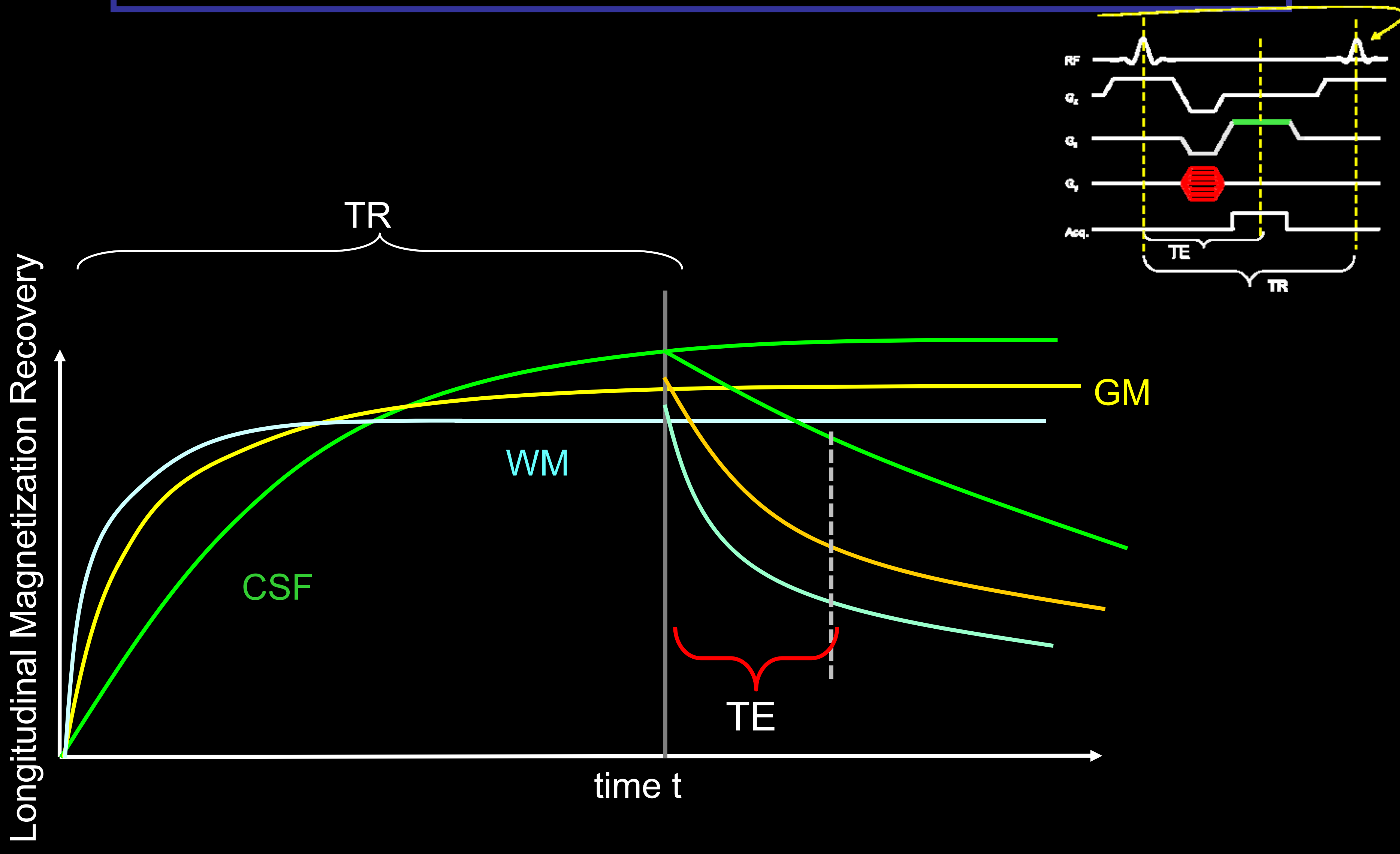


$$TR_{opt} = \frac{\ln\left(\frac{\rho_{0,B}}{T_{1,B}}\right) - \ln\left(\frac{\rho_{0,A}}{T_{1,A}}\right)}{\frac{1}{T_{1,B}} - \frac{1}{T_{1,A}}}$$

T1 values at 3T
GM : ~1200ms
WM: ~800ms
CSF: ~2-4s

$$= T_{1,A}|_{T_{1,A} \cong T_{1,B}}$$

Contrast

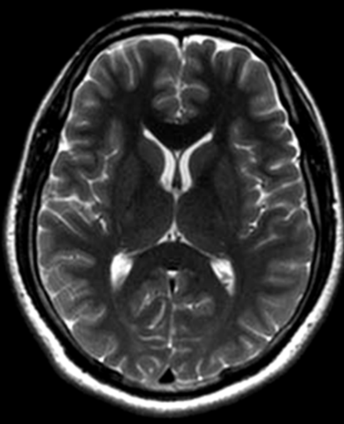


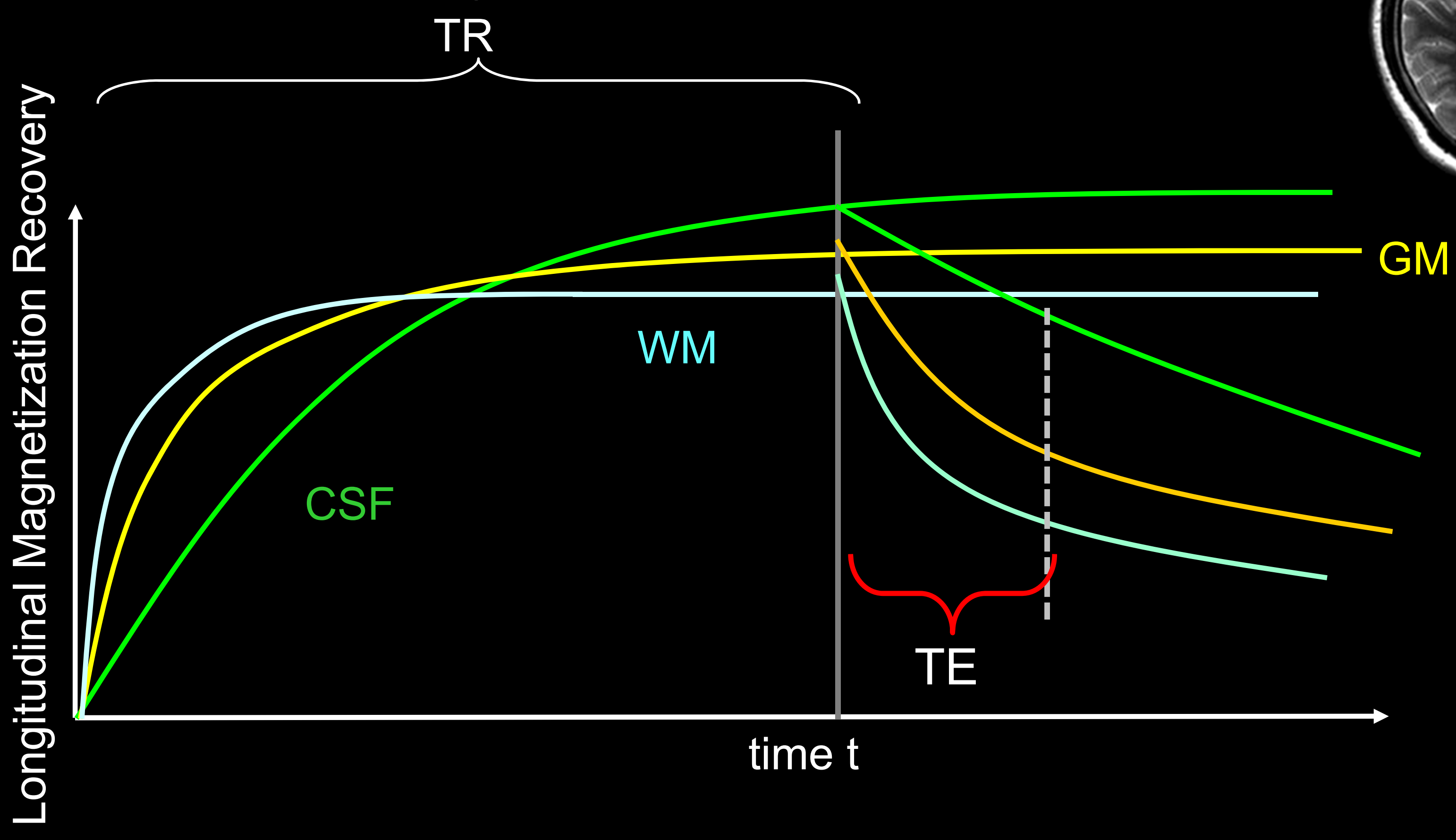
RF
G_z
G_x
G_y
Acq.
TE
TR
TR
GM
WM
CSF
TE
Longitudinal Magnetization Recovery
time t

Contrast…T_2 weighting

Long repeat time - TR

Long Echo time - TE





T2 (T2*) weighting

$$S(TR,TE) = \rho_0(1 - e^{-TR/T_1})e^{-TE/T_2^*}$$

Long TR, Long TE

- **T2* weighting**

use TR>>T1 => e^{-TR/T_1} ->0

$$C_{AB}(TE) = \rho_{0,A} e^{-\frac{TE}{T_{2,A}^*}} - \rho_{0,B} e^{-\frac{TE}{T_{2,B}^*}}$$

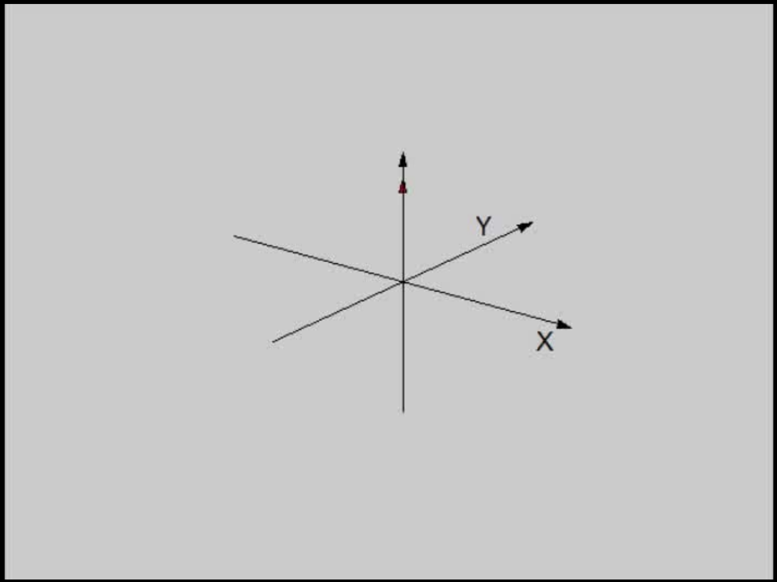
$$TE_{opt} = \frac{\ln\left(\frac{\rho_{0,B}}{T_{2,B}^*}\right) - \ln\left(\frac{\rho_{0,A}}{T_{2,A}^*}\right)}{\frac{1}{T_{2,B}^*} - \frac{1}{T_{2,A}^*}}$$

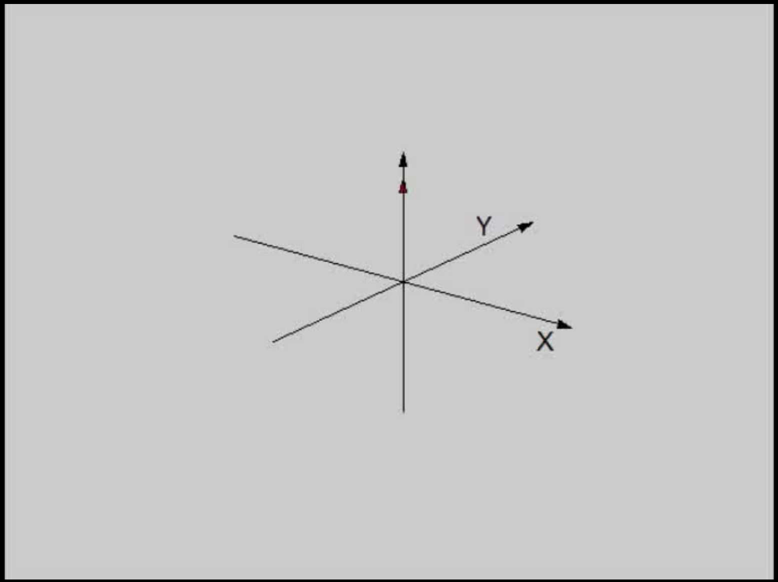
$$= T_{2,A}^*\Big|_{T_{2,A}^* \cong T_{2,B}^*}$$

T1 weighting with inversion recovery (Chap 8.4)

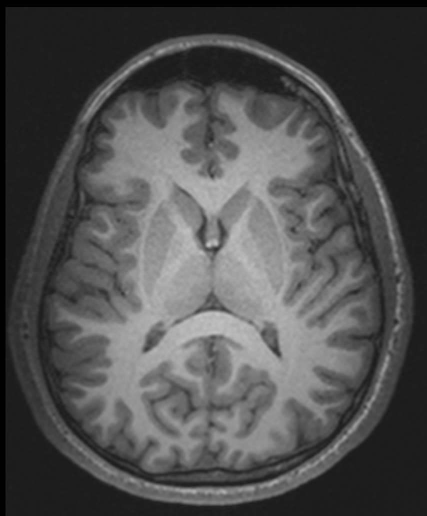
$$M_{\perp}(TI,TE) = M_0(1 - 2e^{-\frac{TI}{T1}})e^{-\frac{TE}{T_2{}^*}}$$

- IR may double the T1 contrast relative to GE
- Optimal TI (?)
- Potential of nulling certain tissues

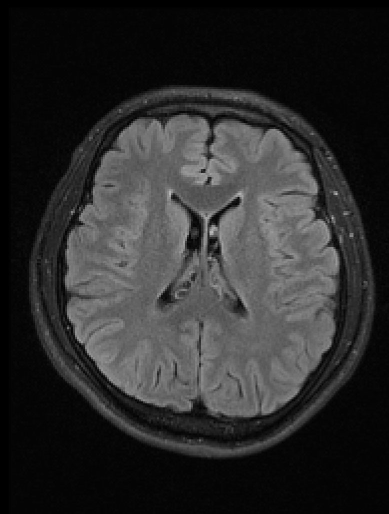




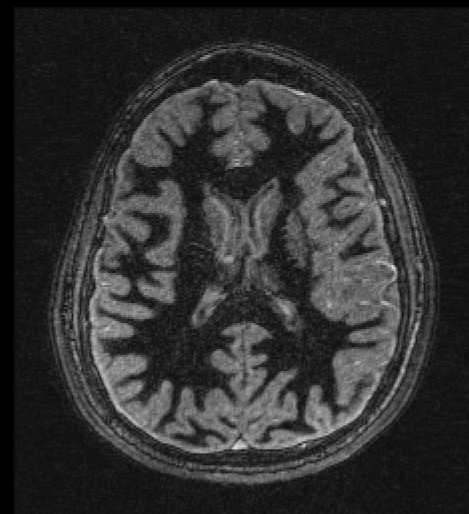
IR examples



T1W



CSF nulling, T2W



WM+CSF nulling

GM+CSF nulling

T2W

Fat nulling,T2W

Contrast enhancement with T1-shortening agents (Gd)

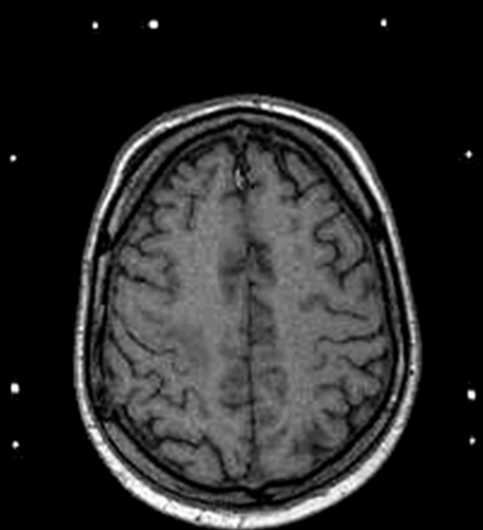
- With T1 shortened , target tissue will have increased signal in short TR GE images
- T1 shortening effect proportional to agent concentration
- Target tissue example:
  - Blood (MR angiography, perfusion weighted imaging)
  - Tumor
  - Vessel wall lesion

$$\frac{1}{T_{1eff}} = \frac{1}{T_1} + \alpha_1[c]$$

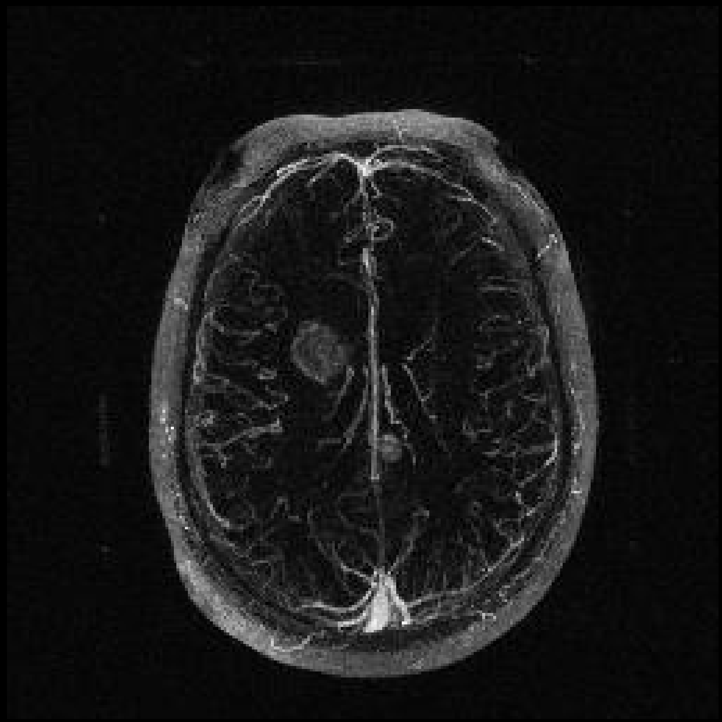
$$\frac{1}{T_{2eff}} = \frac{1}{T_2} + \alpha_2[c]$$

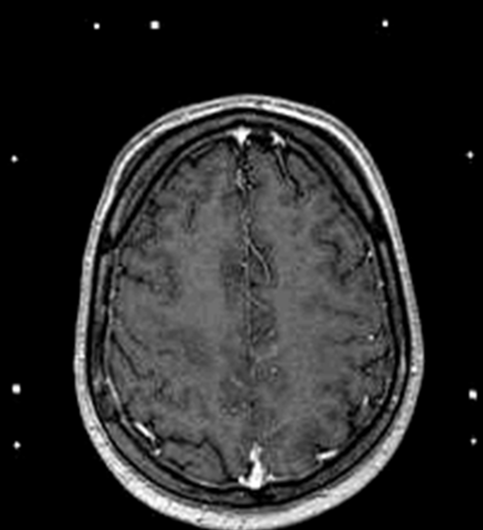
Angiography

Exogenous contrast agents



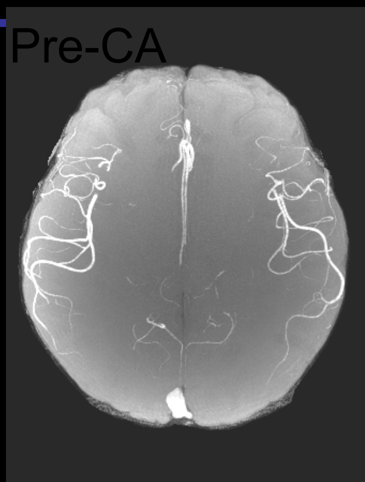
Pre-contrast

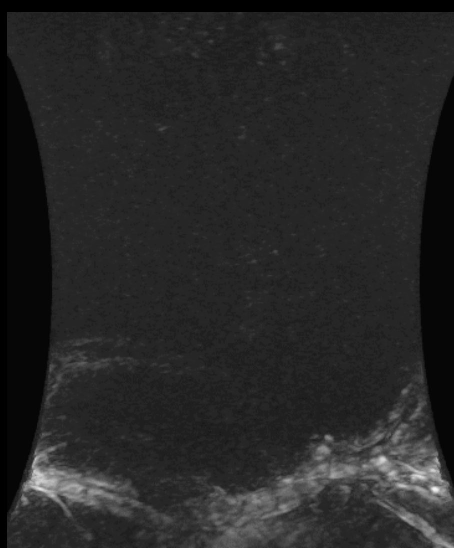


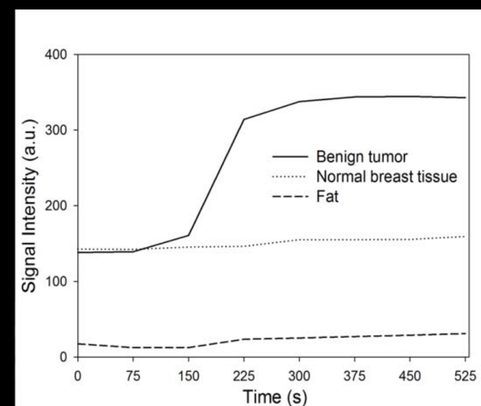


Post-contrast

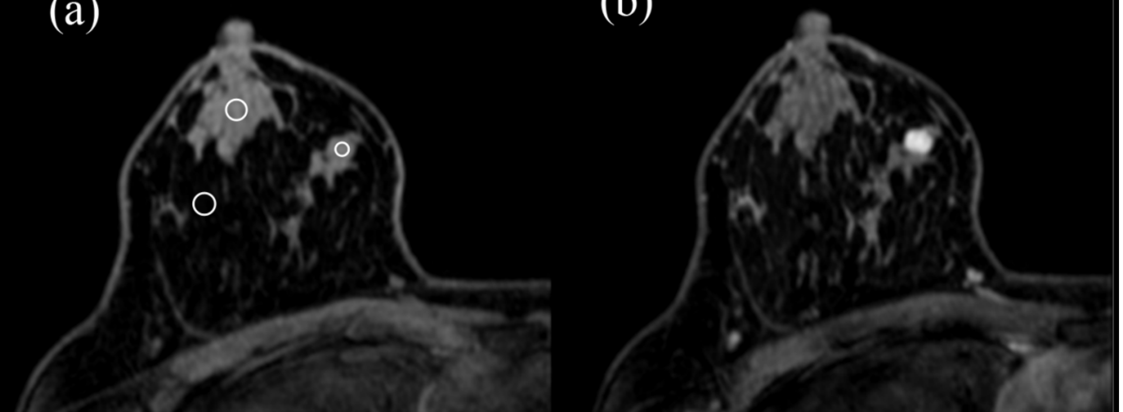
Contrast enhanced images







(a) (b)



CNR, partial volume and resolution

$$\mathcal{V}_{AB} \equiv \frac{C_{AB}}{\sigma_{eff}} = \frac{C_{AB}}{\sigma_0}\sqrt{N_{acq}} = \frac{C_{AB}}{\sigma_0}\sqrt{n_{voxel}}$$

- **For objects occupying multiple voxels, CNR is independent of resolution**

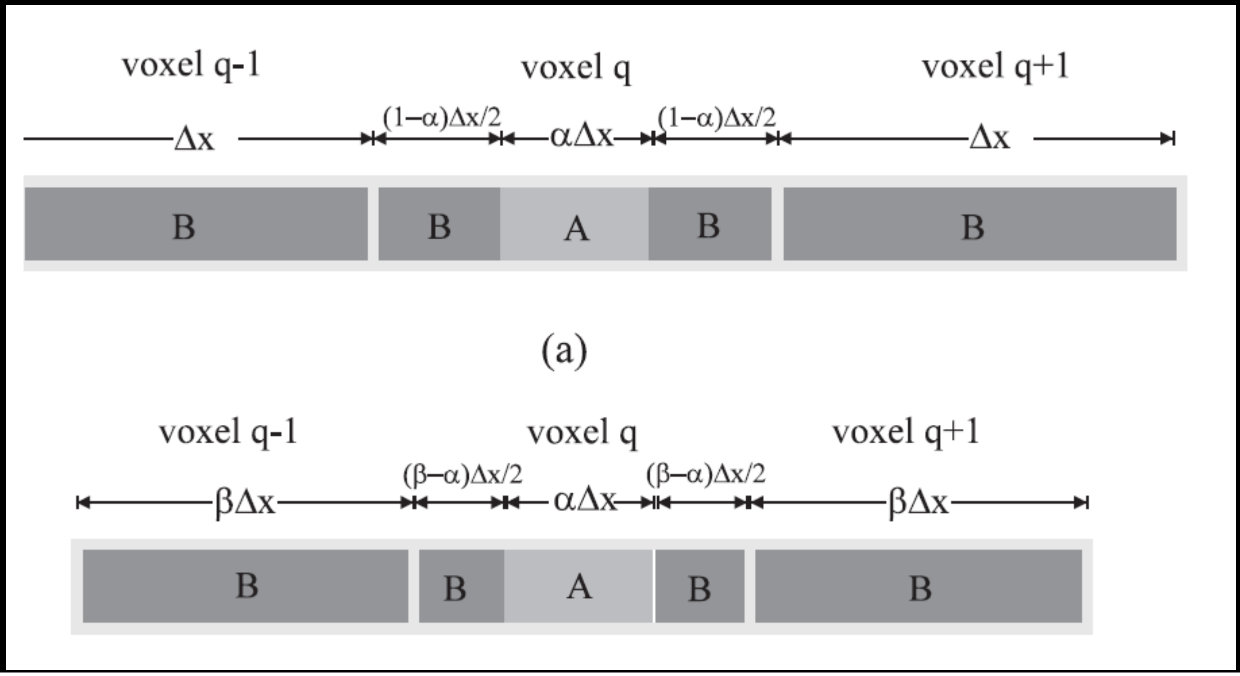
- **Partial volumed voxel contains >1 tissues/objects**

- **Partial volume effect (PVE) is inevitable in in-vivo MR imaging**
  - **Finite voxel size (0.25mm ~ 4mm)**
  - **Complex tissue boundaries**
  - **Micro vasculature/structures**
  - **…**

CNR, partial volume and resolution

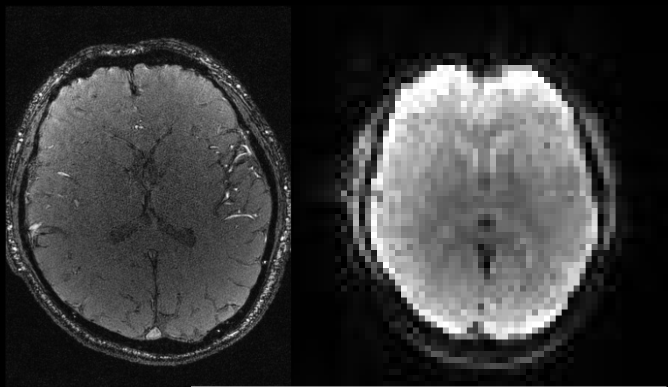
- **Size of A: $\alpha\Delta x$, (α<1)**
- **Size of B: Δx or $\beta\Delta x$,   (β<1)**
- **In both scenarios: $C_{AB} = \alpha(S_A - S_B)$**
- **Noise σ:** $\sigma(\beta) = \sigma_0$,   **(βL_x, N)**

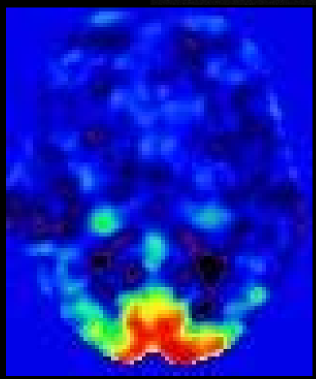
$$\sigma(\beta) = \sqrt{\beta} \cdot \sigma_0, \quad (L_x, N/\beta)$$

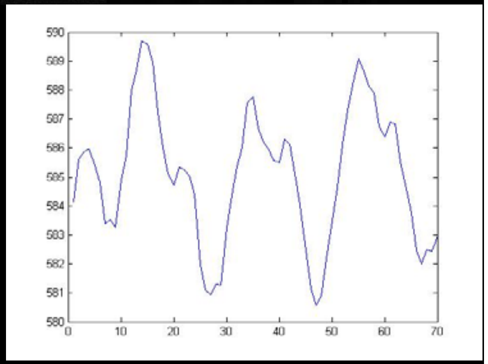


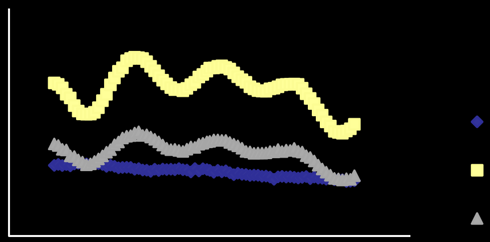
How PV affects the image?

- **Blurring**
- **Signal dephase (T2'/T2*, BOLD imaging, chap 25)**
- **Double/multiple T2/T2* exponential decay**
- **Signal cancellation (water/fat)**

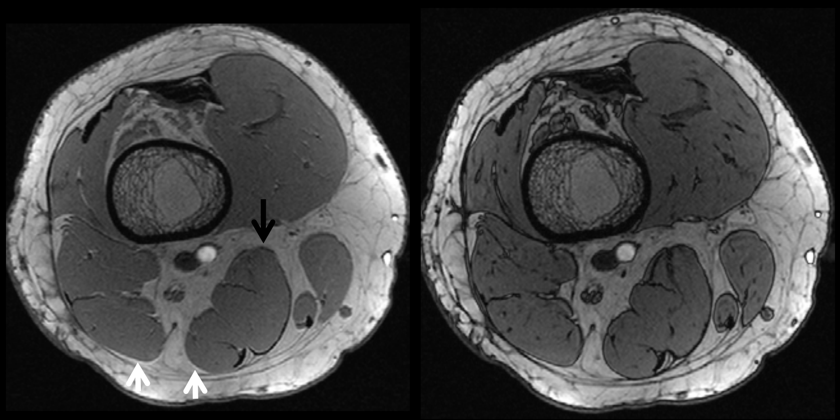








Note the similarity with Fig. 17.8
Courtesy of Dr. Manju Liu



Signal and noise of phase images

- $\sigma_{phase} = \frac{1}{SNR_{mag}}$

- S_{phase} is affected by chemical shift, local susceptibility and/or motion, offering different contrast than magnitude images

- S_{phase} is a relative value since baseline $\phi_0$is unknown

- Contrast in phase has useful information and may still provides sufficient contrast when magnitude fails

Phase imaging applications

- Chemical shift (fat/water) imaging (Chap. 17)
- B_0 field homogeneity mapping (Chap. 20)
- Phase contrast MR angiography (Chap. 24)
- Blood flow quantification (Chap. 24)
- Susceptibility Weighted Imaging (SWI, Chap. 25)
- …

Homework

- Prob 15.1, 15.3, 15.4, 15.8, 15.10, 15.12, 15.13

Next Session

Chapter 15.3-15.6