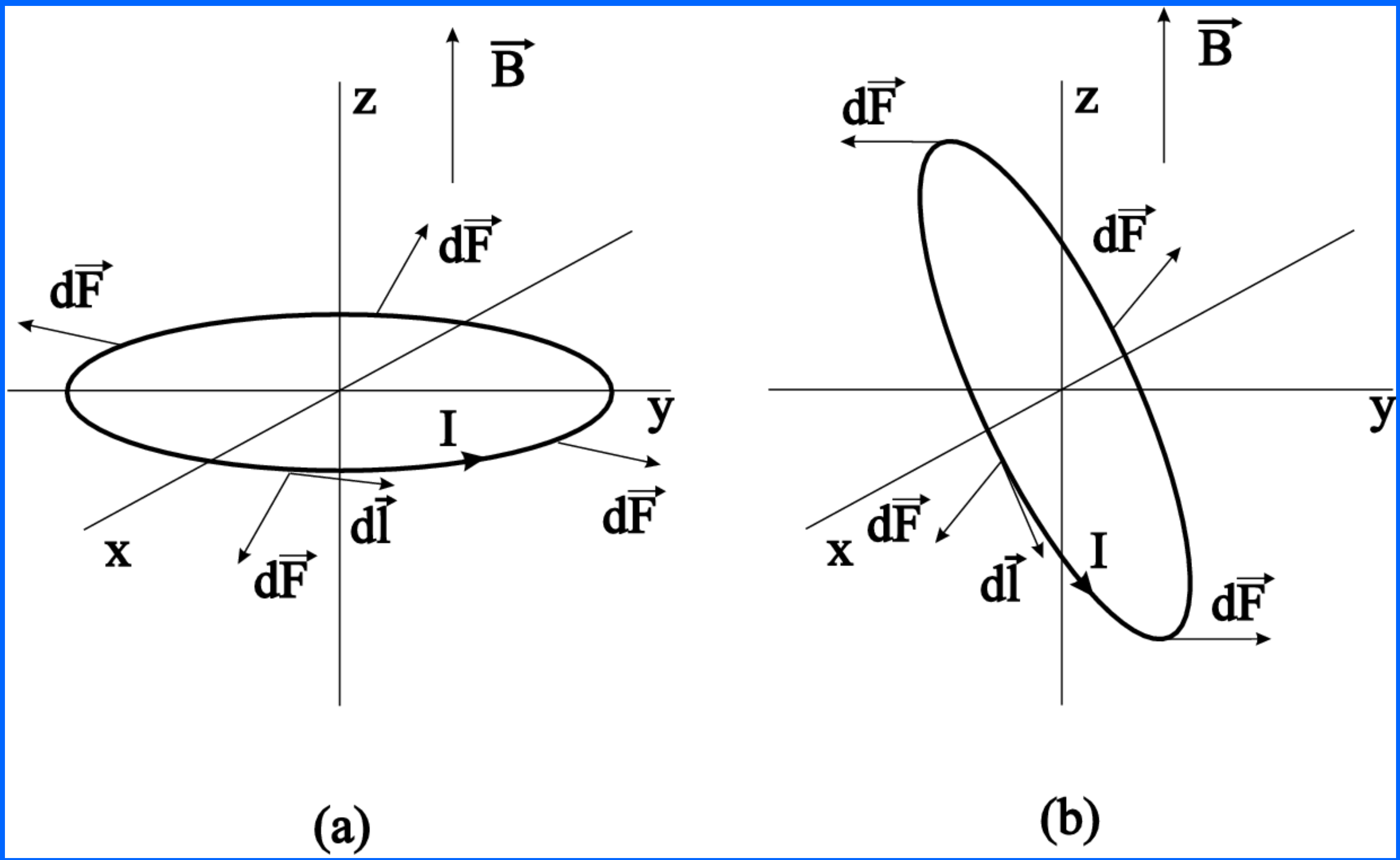
Ch. 2

- Equation of motion: $d\mu/dt = \gamma\mu \times \mathbf{B}$
  This is the simplest version of the Bloch equation.
- Phase: $\phi = -\omega t$
- Larmor equation: $\omega = \gamma B$
- Complex notation: $\mu_+(t) = \mu_+(0) \exp(-i\omega t)$
- Precession
- Gyromagnetic ratio: $\gamma/2\pi = 42.58$ MHz/T

Magnetic force



$d\mathbf{F} = I\, d\mathbf{l} \times \mathbf{B}$

Total force on a closed loop is zero:

$\int d\mathbf{F} = \int I\, d\mathbf{l} \times \mathbf{B} = -\, I\, \mathbf{B} \times \int d\mathbf{l} = 0$

Torque and rotation

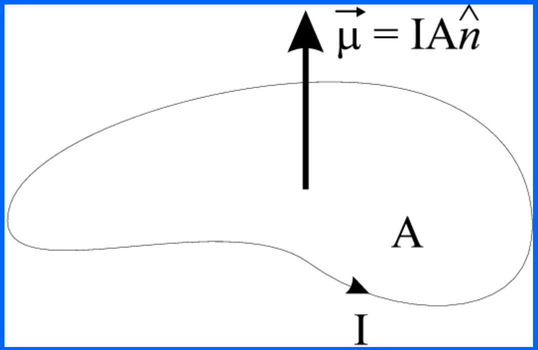
- Even if the net force on a loop is zero, the loop may still rotate.
- Whether a loop will rotate depends on the torque on the loop.
- The torque is defined by $d\mathbf{N} = \mathbf{r} \times d\mathbf{F}$
- In the previous cases, the net torque of the left loop is zero so the loop will not rotate. However, the right loop has a non zero torque.

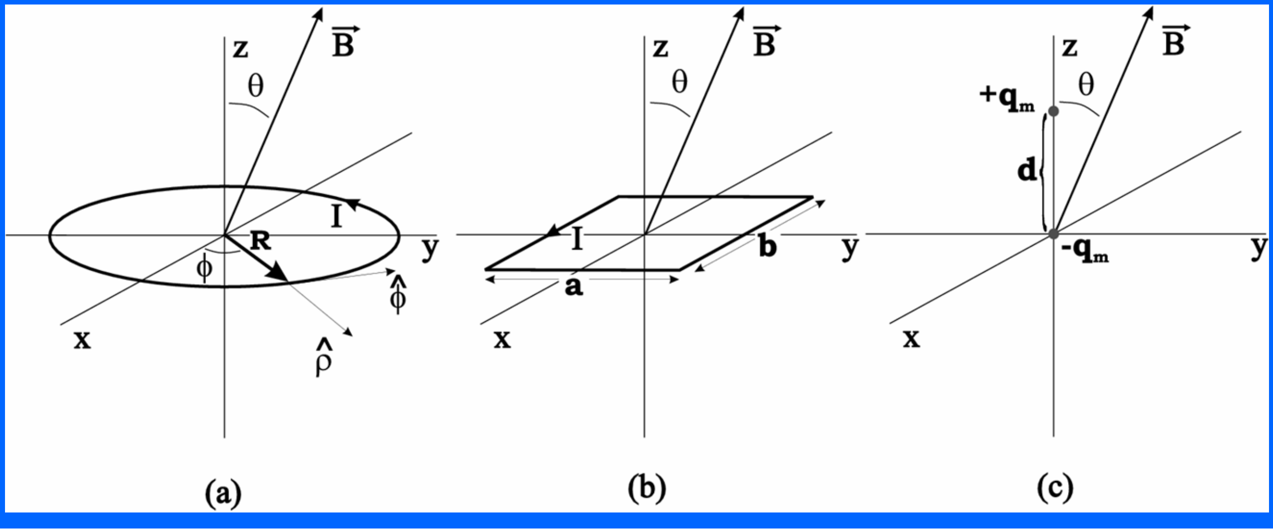
Torque and magnetic moment

- $\mathbf{N} = \int d\mathbf{N} = \int \mathbf{r} \times d\mathbf{F} = \int \mathbf{r} \times (I\, d\mathbf{l} \times \mathbf{B}) = \int (\mathbf{B} \cdot \mathbf{r})\, I\, d\mathbf{l} - \int I\, \mathbf{B}\, (\mathbf{r} \cdot d\mathbf{l}) = \mu \times \mathbf{B}$ (use Stoke's theorem and consider spatially independent I and **B**)
- Where μ is the magnetic moment of the current loop and $\mu = I\, \mathbf{A} = I \int d\mathbf{S}$
- A circular loop example (a): $\mathbf{B} = B(\mathbf{z} \cos\theta + \mathbf{y} \sin\theta)$

  $\rho = \mathbf{x} \cos\phi + \mathbf{y} \sin\phi,\ \phi = -\mathbf{x} \sin\phi + \mathbf{y} \cos\phi \Rightarrow$

  $d\mathbf{N} = IBR^2 \sin\theta \sin\phi\, \phi\, d\phi$ and $\mathbf{N} = -I\pi BR^2 \sin\theta\, \mathbf{x}$





Torque and angular momentum

- Angular momentum **J** = **r** x **p** = **r** x (m**v**)
- d**J**/dt = d**r**/dt x m**v** + **r** x d**p**/dt = 0 + **r** x **F** = **N** (torque) … problem 2.3
- Experiments show $\mu = \gamma$**J** and gyromagnetic ratio for proton is $\gamma = 2.67522 * 10^8$ rad/s/T.
- Famous gamma bar is defined as $\gamma/2\pi$ = 42.58 MHz/T.
- Homework problem 2.4 leads to a result from classical physics: $\gamma =$ q/(2m), where q is the particle charge and m is its mass.

Electron imaging

- $m_p/m_e \sim 1836$. It implies the gyromagnetic ratio of electron is on the order of 1000 times larger than the gyromagnetic ratio of proton. In fact, $\gamma_e/\gamma_p \sim 658$.
- Electron imaging needs to deal with huge energy deposited into human body through microwave spectrum. It's a hardware challenge as well.

Other nuclei imaging

- The net spin or angular momentum of the nucleus has to be non-zero for imaging. That is, the net spin from protons and neutrons in a given nucleus has to be non-zero.
- For example, we can't image ^{16}O or ^{12}C but we can image ^{14}N (because it has 7 protons and 7 neutrons).
- Although all γ are about the same order of magnitude, the abundance of proton is much more than that of any other nucleus in the human body. See Table 2.1.

Equation of motion: simple Bloch equation

- From **N** = μ x **B**, **N** = d**J**/dt, and $\mu = \gamma$**J**, one can easily show $d\mu/dt = \gamma\mu \times$ **B**
- This equation indicates that the magnitude of μ ($|\mu|$) is a constant

Spin motion: Bloch equation

- Clockwise precession: $d\mu = \gamma\mu \times \mathbf{B}\, dt$
- The Larmor frequency

$$\omega_0 = \gamma B$$

- ϕ (phase) $= -\omega t$
- For a proton,

$\gamma =$ gyromagnetic ratio

$= 2\pi \cdot 42.58$ MHz/T

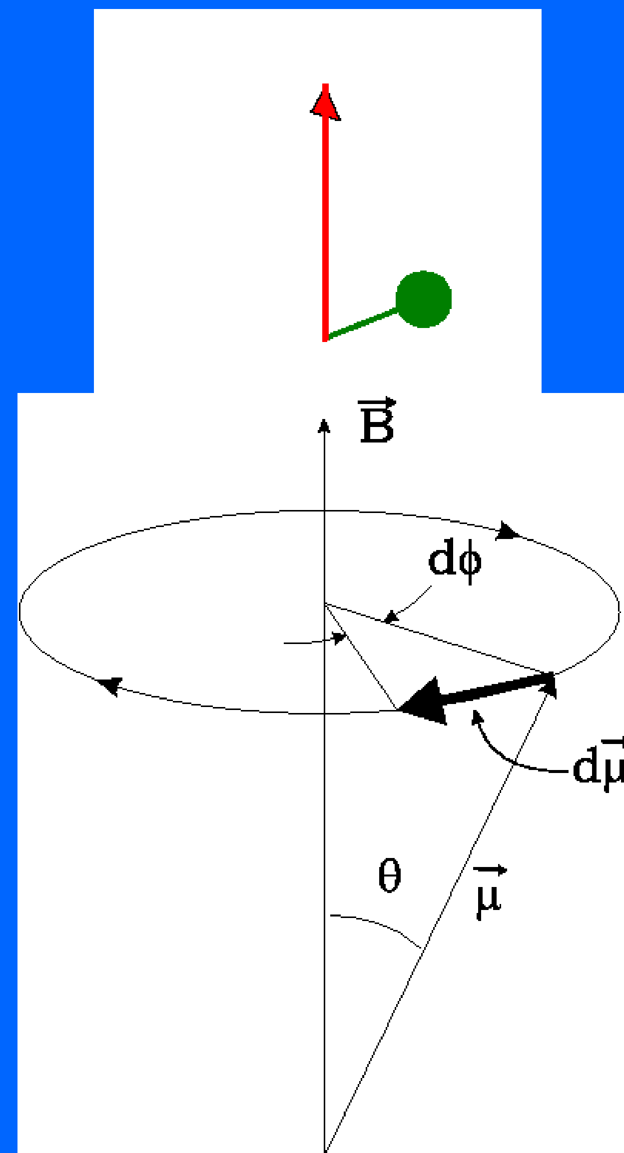


fig from Haacke et al (1999)

Solution to the eq. of motion

- $d\mu/dt = \gamma\mu \times \mathbf{B}$
- When $\mathbf{B} = B_0\, \mathbf{z}$, the solution is $\mu_z(t) = \mu_z(0)$,

$\mu_x(t) = \mu_x(0) \cos(\omega_0 t) + \mu_y(0) \sin(\omega_0 t)$,

$\mu_y(t) = \mu_y(0) \cos(\omega_0 t) - \mu_x(0) \sin(\omega_0 t)$

- Solve the equation in complex representation: Let $\mu_+(t)$ ¼ $\mu_x(t) + i\, \mu_y(t)$, then

  $d\mu_+/dt = -i\, \omega_0\, \mu_+ \Rightarrow \mu_+(t) = \mu_+(0)\, e^{-i\omega_0 t}$
- So the phase ϕ (phase) $= -\omega_0 t$

Ch. 3

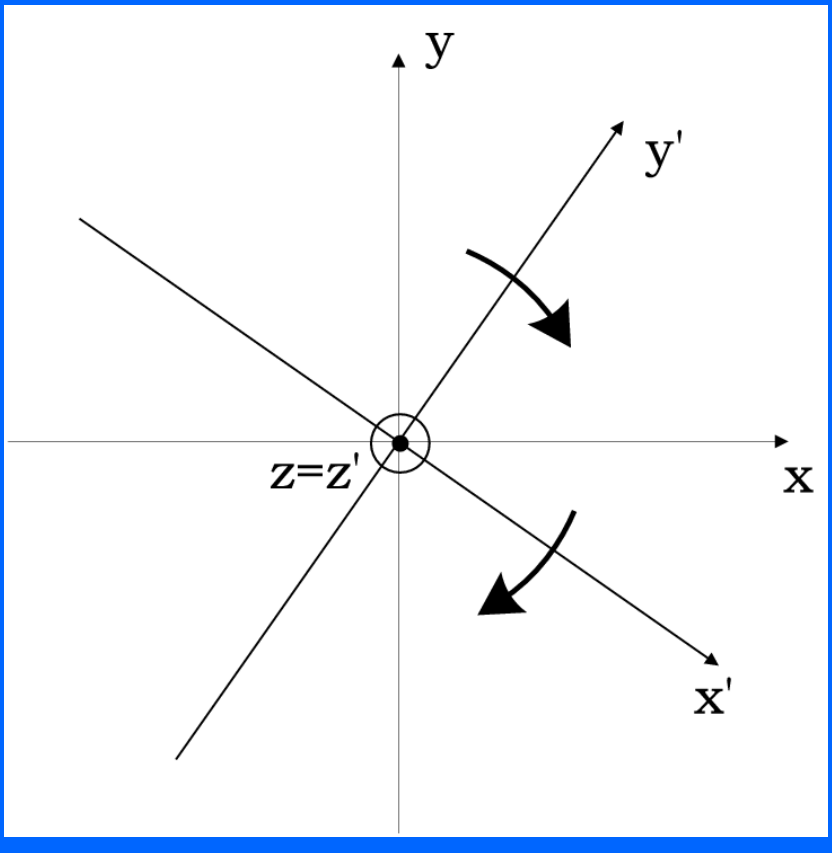
- Equation of motion in the rotating frame

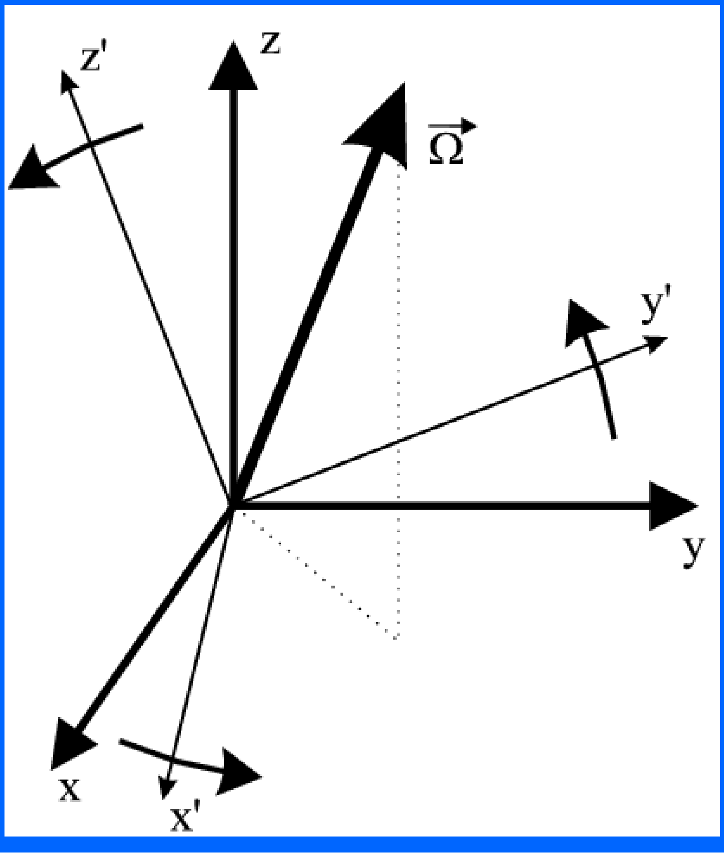
  $(d\mu/dt)' = \gamma\mu \times \mathbf{B}_{eff}$
- Rotating frame vs lab frame
- Effective field

  $\mathbf{B}_{eff} = \mathbf{B} + \Omega/\gamma + \mathbf{B}_1$
- On-resonance behavior
- RF transmit coil

Rotating frame

- Why discuss this? Need a rotating RF field!
- $(d\mu/dt) = (d\mu/dt)' + \Omega \times \mu \Rightarrow \mathbf{B}_{eff} = \mathbf{B} + \Omega/\gamma$
- Explain viewing difference between rotating and lab frames





Rotating frame for an RF field

- The RF field is the so-called B_1 field.
- It's called the transmit RF field in MRI.
- If the RF field is a constant field, a spin will only precess around the vector sum of the B_0 and B_1 fields. It does not help us in MR in obtaining the signal (images).
- Thus, it is natural to have a rotating RF field.
- When the RF field rotates at the Larmor frequency, i.e., γB_0, it is called "on-resonance." Otherwise, it is called "off-resonance."

Linear and quadrature B_1

- Linearly polarized B_1 field = $b_1 \cos(\omega t)$ **x** in the lab frame
- With $\mathbf{x} = \mathbf{x'} \cos(\omega t) + \mathbf{y'} \sin(\omega t)$ and
  $\mathbf{y} = -\mathbf{x'} \sin(\omega t) + \mathbf{y'} \cos(\omega t)$
- $b_1 \cos(\omega t)\, \mathbf{x} =$
  $b_1\, ((1+\cos(2\omega t))\, \mathbf{x'} + \sin(2\omega t)\, \mathbf{y'})/2$ in the rotating frame.
- When the field is averaged over time, only half of the amplitude (b_1 /2) is available in the rotating frame.

Linear and quadrature B_1 (continue)

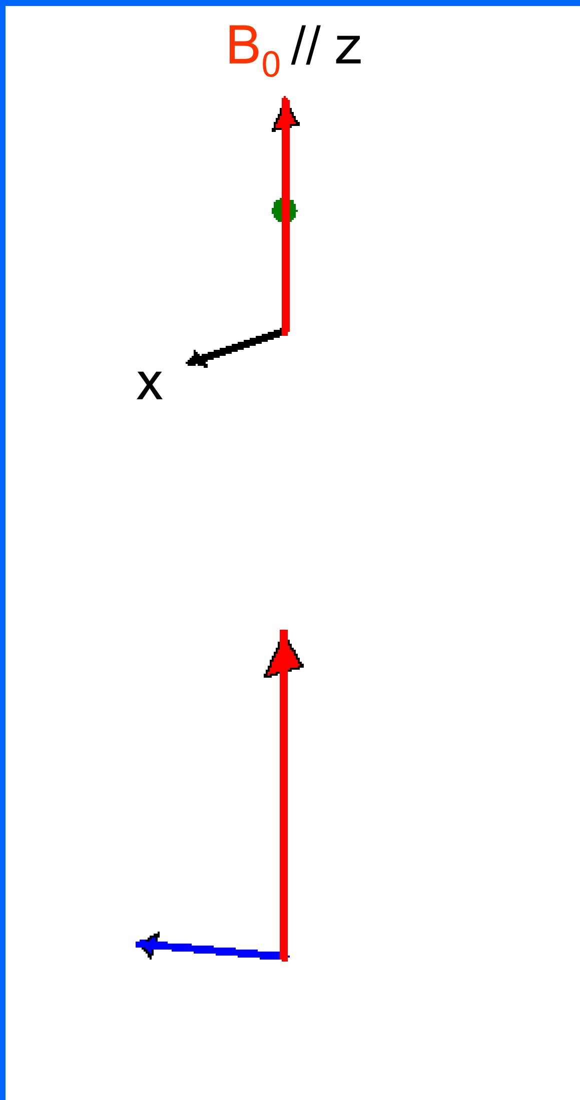
- The quadrature B_1 field is
  $b_1 \mathbf{x'} = \mathbf{x} \cos(\omega t) - \mathbf{y} \sin(\omega t)$
  resting in the rotating frame and it is obvious that its full amplitude is available in the rotating frame.
- Thus, with given RF power, the quadrature field is more efficient than the linear field.

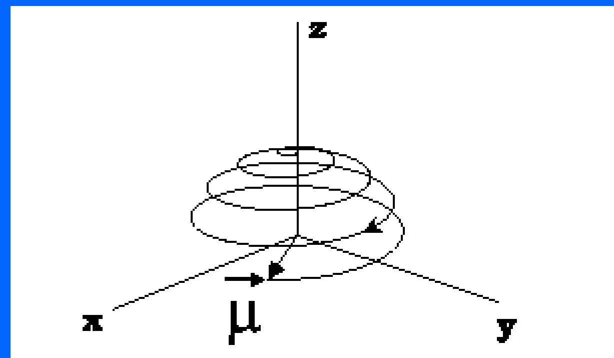
Resonance condition

- $(d\mu/dt)' = \gamma\mu \times \mathbf{B}_{eff}$
- $\mathbf{B}_{eff} = \mathbf{B}_0 + \Omega/\gamma + \mathbf{B}_1 = [(\omega_0 - \omega)\mathbf{z'} + \omega_1\mathbf{x'}]/\gamma$
- When $\omega_0 = \omega$, i.e., on-resonance condition, $\gamma\mathbf{B}_{eff} = \gamma B_1\,\mathbf{x'} = \omega_1\,\mathbf{x'}$
- If B_1 is a constant and the rf pulse is applied for a period of time τ, the flip angle $\theta = \gamma B_1\tau$, However, B_1 usually depends on time, so the general solution of the flip angle is

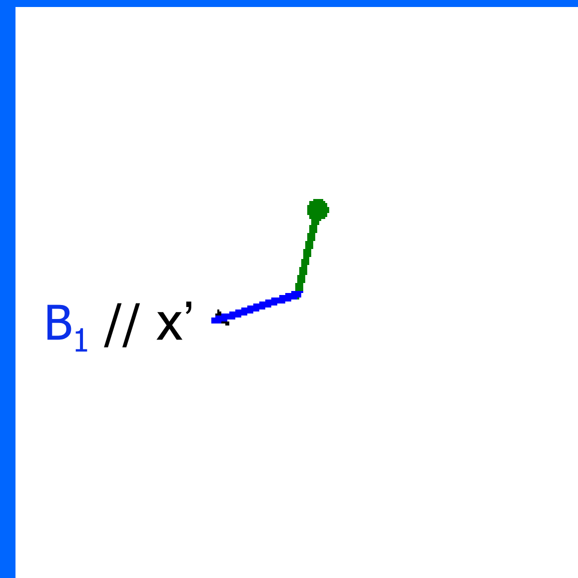
  $\theta = \gamma \int B_1\, dt$
- The position of the spin can be found from solving the Bloch equation. Sec. 3.3.2 provides a simple case (which is solved by rotational matrices).

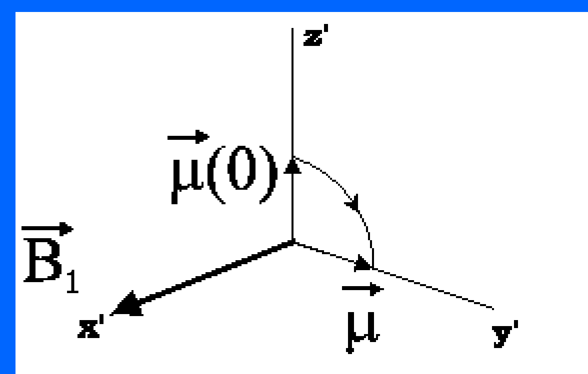
Laboratory vs rotational frame





- Tip the spin in order to detect an MR signal.
- Let the rf field (CP) rotate at the same Larmor frequency as the spin precesses around B_0.
- In the rotational (primed) frame, the spin simply rotates around the x' axis.





Effective left-circularly polarized field

- **x'** ¼ $\mathbf{x}^{left} = \mathbf{x}\cos(\omega t) - \mathbf{y}\sin(\omega t)$
- $\mathbf{x}^{right}$ ¼ $\mathbf{x}\cos(\omega t) + \mathbf{y}\sin(\omega t)$
  $= \mathbf{x'}\cos(2\omega t) + \mathbf{y'}\sin(2\omega t)$
- This means that the right-circularly polarized field is completely ineffective, as its time average is zero.

Off resonance condition

- $\mathbf{B}_{eff} = [(\omega_0 - \omega)\mathbf{z'} + \omega_1\mathbf{x'}]/\gamma$
- Off-resonance: ω_0 û ω
- $\omega_{eff} = [(\omega_0 - \omega)^2 + \omega_1{}^2]^{1/2}$
- Eq. 3.54 can be derived in an easier fashion rather than the given hint.