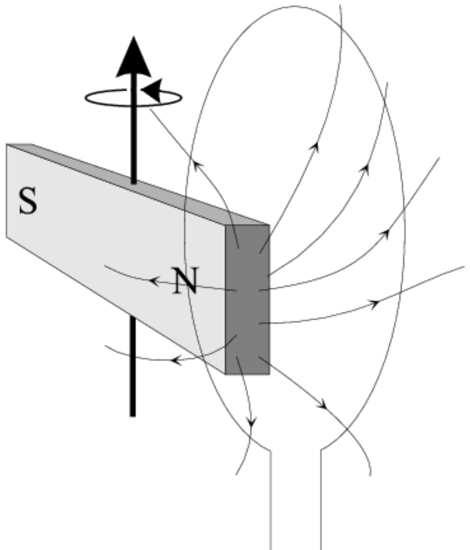
Ch. 7

- Flux
- Reciprocity
- Complex signal behavior
- Demodulation
- Relative signal strength

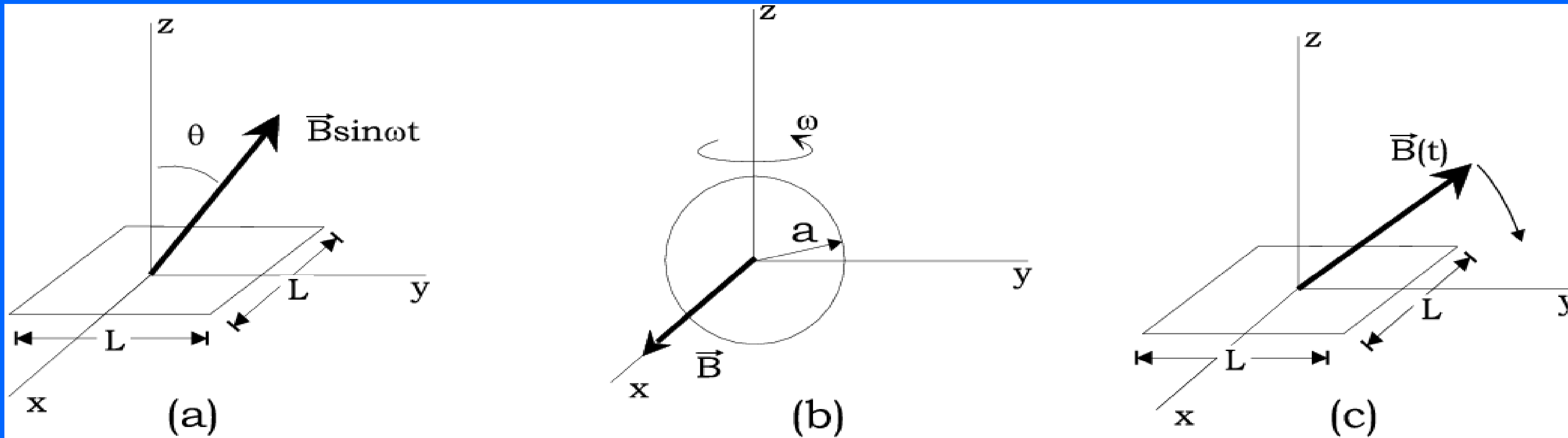
Faraday induction

- Faraday's law of induction: electromotive force (or induced voltage) emf = $-d\Phi/dt$
- Flux $\Phi = \int \mathbf{B} \cdot d\mathbf{S}$ (where **S** represents the coil area).
- A constant **B** field does not produce emf.
- Only time varying **B** field or area changing with time can produce emf.



(a)

Examples



- emf in (a) is $-L^2 B\ \omega \cos\theta \cos \omega t$
- The induced current in (a) flows clockwise when $\sin \omega t > 0$. This can be determined by the direction of the B field.
- emf ó ω => emf increases with frequency.
- Case (c) is close to an MR experiment.

Reciprocity

- Current on a given loop will produce B field.
- On the contrary, given the identical B field distribution in space and time, and provided the same loop, the same current can be considered to be induced. This is reciprocity.
- $\text{emf} = -d\Phi/dt = -d/dt \int d^3r\, (\mathbf{B}(r)/I) \cdot \mathbf{M}(r,t)$ where $\mathbf{B}(r)/I$ only depends on RF coil geometry.
- $\mathbf{M}(r,t)$ is the magnetization.

MR signal

- Signal ó emf = $-d\Phi/dt$
  $= -d/dt \int d^3r\, (\mathbf{B}(r)/I) \cdot \mathbf{M}(r,t)$
  $= -d/dt \int d^3r\, [(B_x M_x + B_y M_y)/I + B_z M_z/I]$
  $= -d/dt \int d^3r\, [B_a M_a\, e^{-t/T_2} \cos(\omega_0 t - \phi_0 + \theta_b)$
  $+ B_z M_z/I\,]$
  $\sim \omega_0 \int d^3r\, B_a M_a\, e^{-t/T_2} \sin(\omega_0 t - \phi_0 + \theta_b)$
- If there exists background inhomogeneous field, replace ω_0 by $\omega_0 + \Delta\omega$ (However, note $|\Delta\omega| << \omega_0$)

Signal demodulation

- Signal ó $\omega_0 \int d^3r\, B_a M_a\, e^{-t/T_2}\, \mathrm{Im}\, e^{i(\theta_b - \phi_0) + i\omega_0 t}$
- How to demodulate: Multiply the signal by $\sin(\omega_0+\delta\omega)t$ (real channel) or $-\cos(\omega_0+\delta\omega)t$ (imaginary channel)
- $\sin(\omega_0 t)\sin(\omega_0+\delta\omega)t = (\cos(\delta\omega t) - \cos(2\omega_0+\delta\omega)t)/2$
- $-\sin(\omega_0 t)\cos(\omega_0+\delta\omega)t = (\sin(\delta\omega t) - \sin(2\omega_0+\delta\omega)t)/2$
- Filter out the high frequency part (low pass filter)

MR complex signal

- Signal demodulation sin($\delta\omega$t) and cos($\delta\omega$t) leads to signal
  ó $\omega_0 \int d^3r\, B_a M_a\; e^{-t/T_2}\, e^{i(\delta\omega+\phi_0-\theta_b)}$

Space-independent limit

- Signal ó $\omega_0 \int d^3r\, B_a M_a\ e^{-t/T_2}\, e^{i(\delta\omega+\phi_0-\theta_b)}$
- If the sample and the rf field are uniform, then the <u>signal of the entire sample</u> ó $\omega_0\, V_s\, B_a M_a\ e^{-t/T_2}\, e^{i(\delta\omega+\phi_0-\theta_b)}$
- In addition, $M_a \sim M_0 \sin\theta$, where M_0 ó B_0. Thus, signal ó B_0^2.
- However, noise ó B_0, so signal-to-noise ratio (or SNR) ó B_0.

Relative signal strength (R_i)

- Eq. (6.10) shows $M_{0,i}$ ó $a_i s_i(s_i+1)\gamma_i^2$, where a_i is the natural abundance and s_i is the spin of a particular nucleus i.
- Signal ó ω_0 $M_{0,i}$ ó R_i ¼ $r_i a_i s_i(s_i+1)|\gamma_i|^3$ where r_i is the relative abundance in the human body.

RF field effects

- Generally, larger $B_1^{transmit}$ (up to 90º flip angle) or $B^{receive}$ leads to brighter image.
- Smaller $B_1^{transmit}$ or $B^{receive}$ leads to darker image.
- It is important to obtain uniform rf (B_1) fields.
- Signals due to transmitting fields are actually complicated. (See Bloch equations and Ch. 16.)

Independent coils

- Independent coils mean no noise correlation between coils.
- Such setup of coils will improve SNR or can be used to image different parts of the object.
- This is the beginning of parallel imaging.